Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

A-level
MATHEMATICS
Paper 1

Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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11	
12	
13	
14	
15	
TOTAL	

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Answer **all** questions in the spaces provided.

1 A curve is defined by the parametric equations

$$x = \cos\theta \quad \text{and} \quad y = \sin\theta \qquad \text{where } 0 \le \theta \le 2\pi$$

Which of the options shown below is a Cartesian equation for this curve?

Circle your answer.

[1 mark]

$\frac{y}{x} = \tan\theta$ $\qquad$ $x^2 + y^2 = 1$ $\qquad$ $x^2 - y^2 = 1$ $\qquad$ $x^2y^2 = 1$

2 A periodic sequence is defined by

$$U_n = (-1)^n$$

State the period of the sequence.

Circle your answer.

[1 mark]

-1 $\qquad$ 0 $\qquad$ 1 $\qquad$ 2

3 The curve

$$y = \log_4 x$$

is transformed by a stretch, scale factor 2, parallel to the y-axis.

State the equation of the curve after it has been transformed.

Circle your answer.

[1 mark]

$y = \frac{1}{2}\log_4 x$ $\qquad$ $y = 2\log_4 x$ $\qquad$ $y = \log_4 2x$ $\qquad$ $y = \log_8 x$

Do not write outside the box

4 The graph of

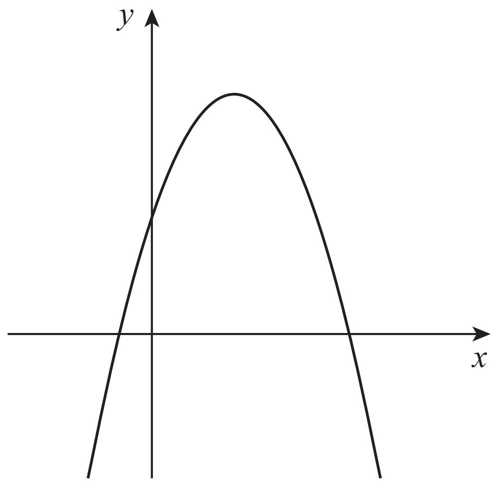
$$y = \mathrm{f}(x)$$

where

$$\mathrm{f}(x) = ax^2 + bx + c$$

is shown in **Figure 1**.

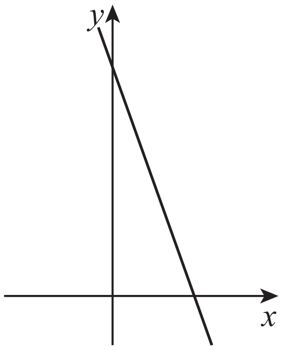
Figure 1

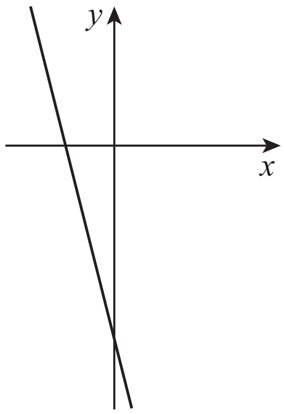


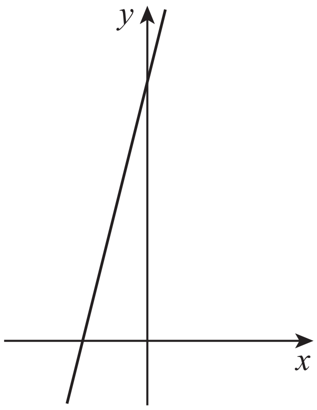
Which of the following shows the graph of $y = \mathrm{f}'(x)$?

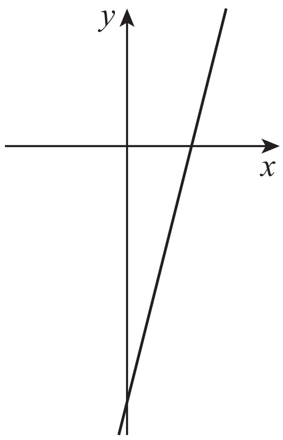
Tick (✓) **one** box.

[1 mark]









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5 Find an equation of the tangent to the curve

$$y = (x - 2)^4$$

at the point where $x = 0$

[3 marks]

$\frac{dy}{dx} = 4(x-2)^3$ at $x = 0$ $\frac{dy}{dx} = -32$

at $x = 0$ $y = 16$ $y = -32x + c$

$16 = c$

$y = -32x + 16$

Do not write outside the box

6 (a) Find the first two terms, in ascending powers of x, of the binomial expansion of

$$\left(1 - \frac{x}{2}\right)^{\frac{1}{2}}$$

[2 marks]

$= 1 + \frac{1}{2}\left(\frac{-x}{2}\right) \quad = 1 - \frac{x}{4}$

6 (b) Hence, for small values of x, show that

$$\sin 4x + \sqrt{\cos x} \approx A + Bx + Cx^2$$

where A, B and C are constants to be found.

[4 marks]

Small angles $\sin x = 4x$

$\cos x = 1 - \frac{x^2}{2}$

$4x + \sqrt{1 - \frac{x^2}{2}}$ from part a sub x^2 instead of x

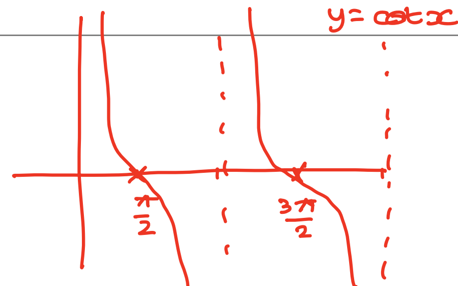
$= 4x + 1 - \frac{x^2}{4}$

$4x + \sqrt{1 - \frac{x^2}{2}} = 1 + 4x - \frac{x^2}{4}$

Turn over ▶

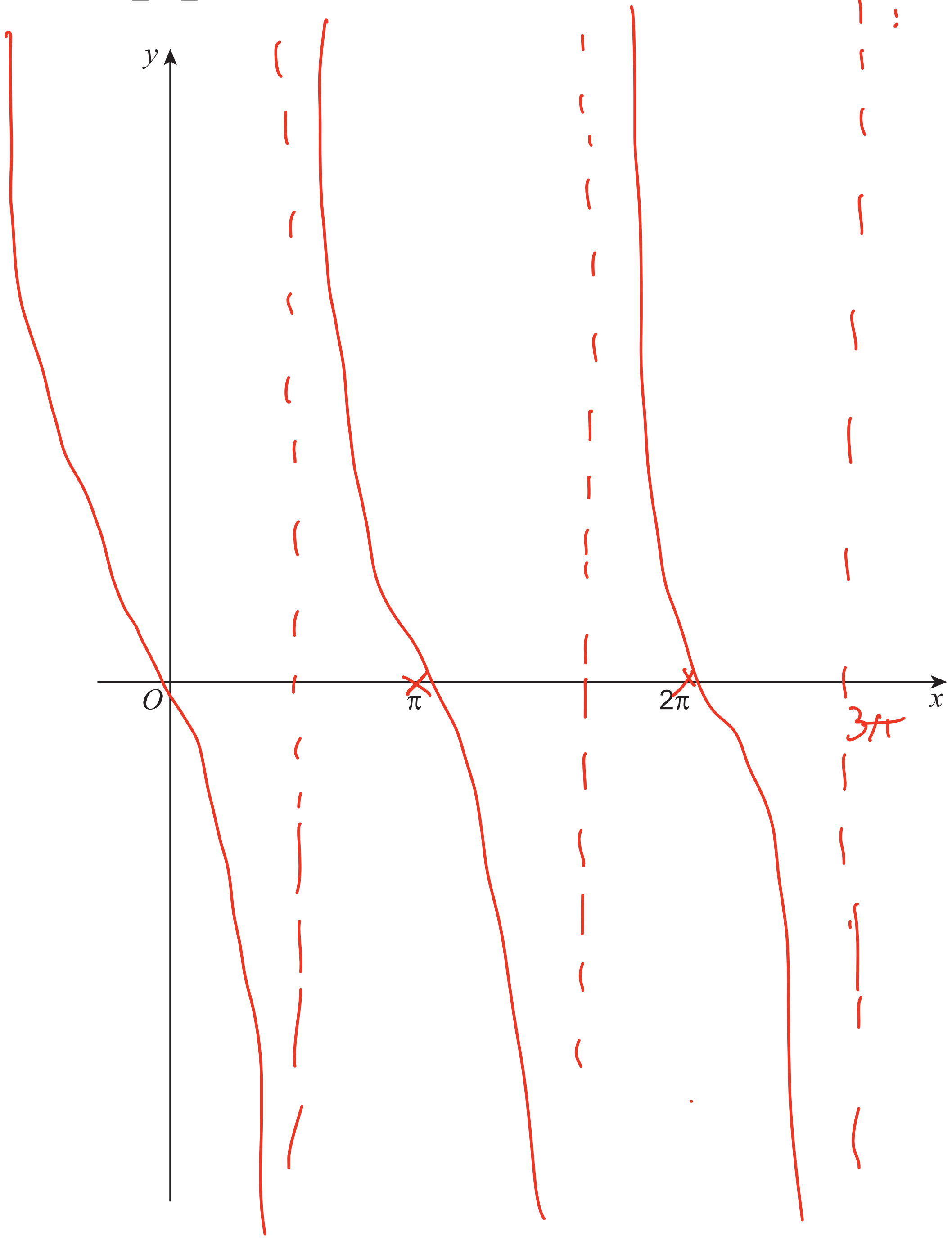
7 Sketch the graph of

$$y = \cot\left(x - \frac{\pi}{2}\right)$$



for $0 \leq x \leq 2\pi$

[3 marks]



8 The lines L_1 and L_2 are parallel.

L_1 has equation

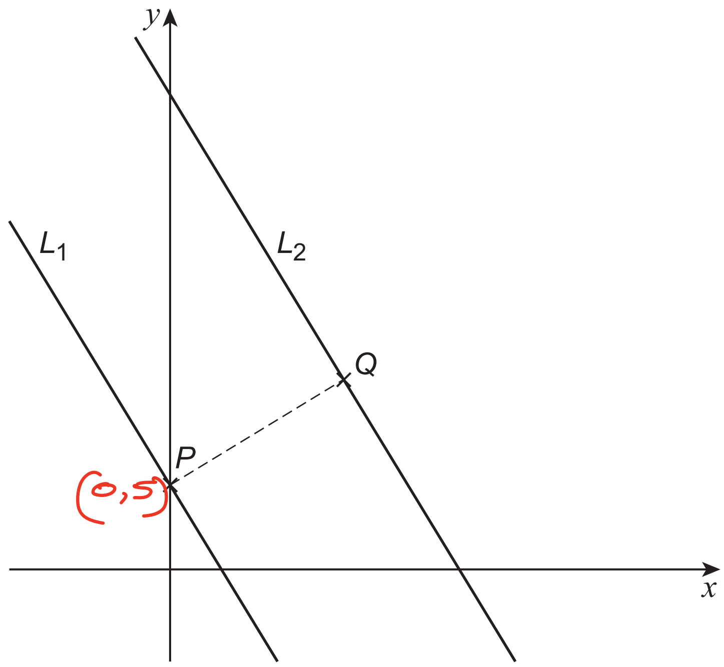
$$5x + 3y = 15$$

and L_2 has equation

$$5x + 3y = 83$$

L_1 intersects the y-axis at the point P.

The point Q is the point on L_2 closest to P, as shown in the diagram.



8 (a) (i) Find the coordinates of Q.

[5 marks]

$5x + 3y = 15$ gradient $PQ = \frac{3}{5}$

$3y = -5x + 15$ at $(0,5)$

$y = \frac{-5}{3}x + 5$ $y = \frac{3}{5}x + c$ $c = 5$

line PQ $y = \frac{3}{5}x + 5$ $5x + 3y = 83$

intersect at

$5x + \frac{9}{5}x + 15 = 83$ $\frac{34}{5}x = 68$

$x = 10$ $y = 11$

$Q\ (10, 11)$

Do not write outside the box

8 (a) (ii) Hence show that $PQ = k\sqrt{34}$, where k is an integer to be found.

[2 marks]

$(0,5)$ $(10,11)$ $PQ = \sqrt{10^2+6^2}$

$= \sqrt{136}$

$PQ = 2\sqrt{34}$

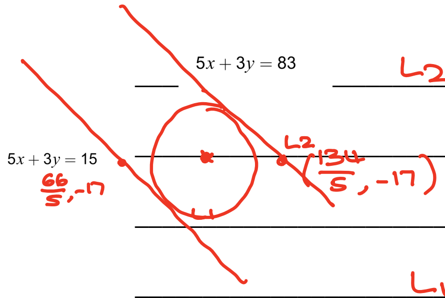
Do not write outside the box

8 (b) A circle, C, has centre $(a, -17)$.

L_1 and L_2 are both tangents to C.

8 (b) (i) Find a.

[2 marks]



L_2 at $y = -17$ $\quad 5x - 51 = 83$

$5x = 134$

$x = \frac{134}{5}$

L_1 at $y = -17$ $\quad 5x - 51 = 15$

$5x = 66$

$x = \frac{66}{5}$

mid point of $\left(\frac{66}{5}, -17\right)$ and $\left(\frac{134}{5}, -17\right)$

$= \left(\frac{100}{5}, -17\right) = (20, -17)$ $\quad a = 20$

8 (b) (ii) Find the equation of C.

[2 marks]

as shortest distance $PQ = \sqrt{34}$ radius $= \sqrt{34}$

equation of circle $(x-20)^2 + (y+17)^2 = 34$

Do not write outside the box

9 The first three terms of an arithmetic sequence are given by

$$2x + 5 \qquad 5x + 1 \qquad 6x + 7$$

9 (a) Show that $x = 5$ is the only value which gives an arithmetic sequence.

[3 marks]

$5x+1-(2x+5) = 6x+7-(5x+1)$

$3x-4 = x+6$

$2x = 10$

$x = 5$

9 (b) (i) Write down the value of the first term of the sequence.

[1 mark]

$2(5)+5 = 15$

9 (b) (ii) Find the value of the common difference of the sequence.

[1 mark]

11

15, 26, 37 $\qquad d = 11$

Do not write outside the box

9 (c) The sum of the first N terms of the arithmetic sequence is S_N where

$$S_N < 100\,000$$

$$S_{N+1} > 100\,000$$

Find the value of N.

[4 marks]

$S_n = \frac{n}{2}(2a + d(n-1))$

$\frac{n}{2}(30 + 11(n-1)) < 100,000$

$\frac{n}{2}(19 + 11n) < 100,000$

$\frac{11}{2}n^2 + \frac{19}{2}n - 100,000 < 0$

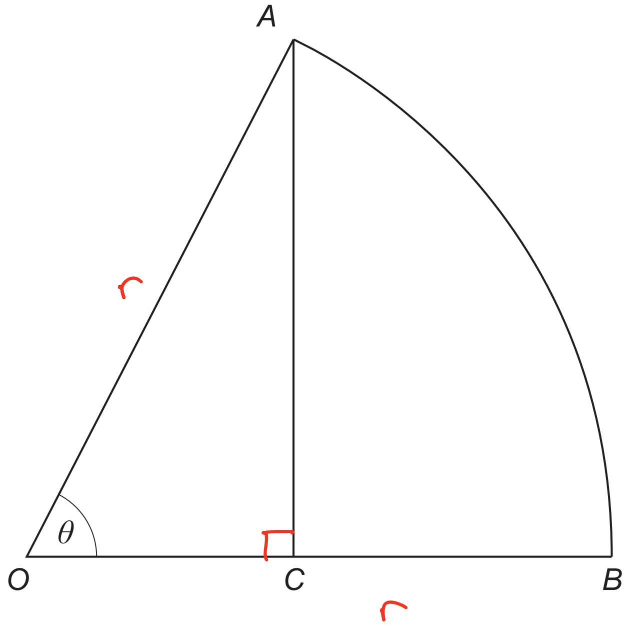
$11n^2 + 19n - 200,000 < 0$ $n = 133$ $S_n = 98553$

$n = 133.9$ (-135.7) invalid $n = 134$ $S_n = 100,031$

$N = 133$

Do not write outside the box

10 The diagram shows a sector of a circle OAB.



The point C lies on OB such that AC is perpendicular to OB.

Angle AOB is θ radians.

10 (a) Given the area of the triangle OAC is half the area of the sector OAB, show that

$$\theta = \sin 2\theta$$

[4 marks]

Area sector $= \frac{\theta}{2\pi} \times \pi r^2 = \frac{\theta r^2}{2}$

Area triangle $= \frac{1}{2} r\sin\theta \times r\cos\theta$

$= \frac{1}{2} r^2 \sin\theta\cos\theta$ $\quad 2\sin\theta\cos\theta = \sin 2\theta$

$= \frac{r^2}{4} \sin 2\theta$

$2 \times \frac{r^2}{4} \sin 2\theta = \frac{\theta r^2}{2}$

$\frac{r^2}{2} \sin 2\theta = \frac{r^2}{2}\theta$ $\quad \sin 2\theta = \theta$

Do not write outside the box

10 (b) Use a suitable **change of sign** to show that a solution to the equation

$$\theta = \sin 2\theta$$

lies in the interval given by $\theta \in \left[\frac{\pi}{5}, \frac{2\pi}{5}\right]$

[2 marks]

$\theta - \sin 2\theta = 0$

$\theta = \frac{\pi}{5}$ $\quad \frac{\pi}{5} - 0.95105 = -0.3227$

$\theta = \frac{2\pi}{5}$ $\quad \frac{2\pi}{5} - 0.5877 = 0.6688$

change of sign hence solution lies between

$\frac{\pi}{5}$ and $\frac{2\pi}{5}$

Question 10 continues on the next page

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10 (c) The Newton-Raphson method is used to find an approximate solution to the equation

$$\theta = \sin 2\theta$$

$\theta - \sin 2\theta$

10 (c) (i) Using $\theta_1 = \frac{\pi}{5}$ as a first approximation for θ apply the Newton-Raphson method twice to find the value of θ_3

Give your answer to three decimal places.

[3 marks]

$x_{n+1} = x_n - \dfrac{(x_n - \sin 2x_n)}{1 - 2\cos 2x_n}$ $\dfrac{d(\theta - \sin 2\theta)}{d\theta}$

$= 1 - 2\cos 2\theta$

$\theta_1 = \dfrac{\pi}{5}$ $\theta_2 = 1.473257\ldots$ θ_3 $1.041324\ldots$

$\theta_3 = 1.041$ (3dp)

10 (c) (ii) Explain how a more accurate approximation for θ can be found using the Newton-Raphson method.

[1 mark]

The more iterations that are used the more accurate the approximations become

10 (c) (iii) Explain why using $\theta_1 = \frac{\pi}{6}$ as a first approximation in the Newton-Raphson method does not lead to a solution for θ.

[2 marks]

$\theta = \frac{\pi}{6}$ $\cos 2\theta = \frac{1}{2}$ which would give a denominator of 0 which would give an undefined solution

Turn over for the next question

Do not write outside the box

11 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + (b+2)x^2 + 2(b+2)x + 8$$

where b is a constant.

11 (a) Use the factor theorem to prove that $(x+2)$ is a factor of $\mathrm{p}(x)$ for **all** values of b.

[3 marks]

if $x+2$ is a factor $f(-2) = 0$

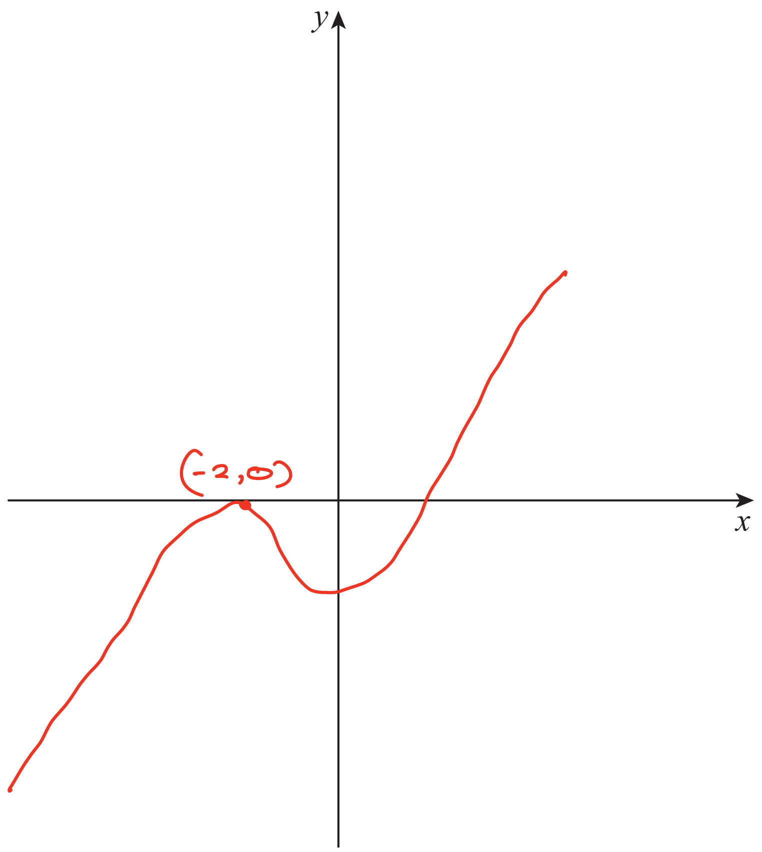
$f(-2) = -8 + 4(b+2) - 4(b+2) + 8$

$= 0$ for all values of b

11 (b) The graph of $y = \mathrm{p}(x)$ meets the x-axis at exactly two points.

11 (b) (i) Sketch a possible graph of $y = \mathrm{p}(x)$

[3 marks]



Do not write outside the box

11 (b) (ii) Given $p(x)$ can be written as

$$p(x) = (x+2)(x^2+bx+4)$$

find the value of b.

Fully justify your answer.

[4 marks]

as there is a repeated root either $(x+2)$ is a repeated root so $x+2$ is a solution of $x^2+bx+4=0$

$4-2b+4=0$ $b=4$

or x^2+bx+4 is a square no

$b^2-4ac=0$

$b^2-16=0$

$b^2=16$ $b=\pm 4$ However if $b=4$ this gives only one root to the cubic $(x+2)(x^2+4x+4)=(x+2)^3$

so b must $=-4$ $(x+2)(x^2-4x+4)$

$(x+2)(x-2)^2$

Hence $b=-4$

Turn over for the next question

Do not write outside the box

12 (a) A geometric sequence has first term 1 and common ratio $\frac{1}{2}$

12 (a) (i) Find the sum to infinity of the sequence.

[2 marks]

$a = 1 \quad r = \frac{1}{2} \qquad S_\infty = \frac{a}{1-r} = \frac{1}{\frac{1}{2}} = 2$

12 (a) (ii) Hence, or otherwise, evaluate

$$\sum_{n=1}^{\infty} (\sin 30°)^n$$

[2 marks]

$\sin 30 = \frac{1}{2} \qquad \frac{1}{2}, + \frac{1}{4} + \frac{1}{8} + \ldots + \ldots$

$= \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots\right) - 1$

$= 2 - 1$

$= 1$

Do not write outside the box

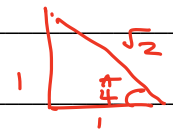
12 (b) Find the smallest positive exact value of θ, in **radians**, which satisfies the equation

$$\sum_{n=0}^{\infty} (\cos\theta)^n = 2 - \sqrt{2}$$

[4 marks]

$a = 1$ $\quad \sum_{0}^{\infty} = \frac{a}{1-r}$ $\quad \frac{1}{1-r} = 2-\sqrt{2}$

$\frac{1}{2-\sqrt{2}} = 1-r$

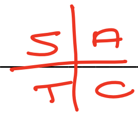


$\frac{1}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{2+\sqrt{2}}{2} = 1-r$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$2+\sqrt{2} = 2-2r$

$2r = -\sqrt{2}$



$r = -\frac{\sqrt{2}}{2}$ $\quad \cos\left(\pi - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\theta = \frac{3\pi}{4}$

Turn over for the next question

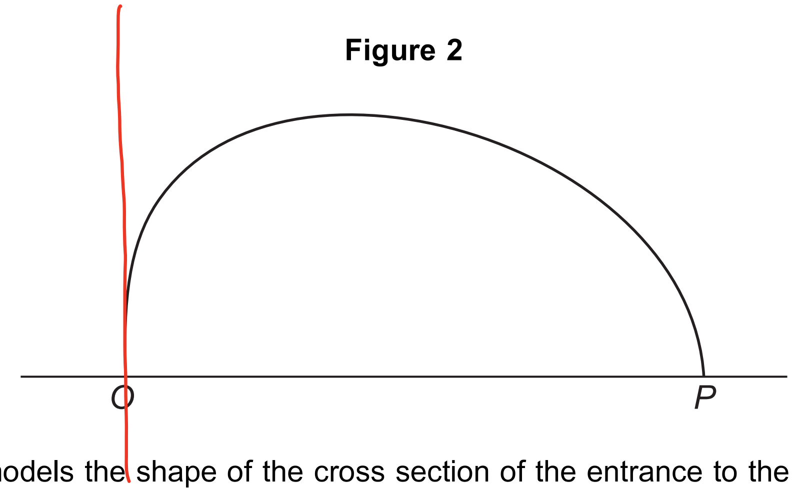
Turn over ▶

Do not write outside the box

13 **Figure 2** shows the approximate shape of the vertical cross section of the entrance to a cave. The cave has a horizontal floor.

The entrance to the cave joins the floor at the points *O* and *P*.

Figure 2



Garry models the shape of the cross section of the entrance to the cave using the equation

$$x^2 + y^2 = a\sqrt{x} - y$$

(0,0) (16,0)

where a is a constant, and x and y are the horizontal and vertical distances respectively, in metres, measured from *O*.

13 (a) The distance *OP* is 16 metres.

Find the value of a that Garry should use in the model.

[2 marks]

at (16,0) $16^2 = a\sqrt{16}$

$256 = 4a$ $a = 64$

(If the axis isn't there is would just involve a shift

along the x axis so a would be the same

Do not write outside the box

13 (b) Show that the maximum height of the cave above *OP* is approximately 10.5 metres. **[6 marks]**

$x^2 + y^2 = a\sqrt{x} - y$

$x^2 - 64x + y^2 + y = 0$ max value when $\frac{dy}{dx} = 0$

$\frac{d}{dx}(x^2 - 64\sqrt{x} + y^2 + y = \frac{d(0)}{dx}$

$2x - 32x^{-\frac{1}{2}} + 2y\frac{dy}{dx} + \frac{dy}{dx} = 0$

$2y + 1\left(\frac{dy}{dx}\right) = \frac{32}{\sqrt{x}} - 2x$

$\frac{dy}{dx} = \frac{\frac{32 - 2x\sqrt{x}}{\sqrt{x}}}{2y+1} = \frac{32 - 2x\sqrt{x}}{\sqrt{x}(2y+1)}$

when $\frac{dy}{dx} = 0$ $\quad 32 - 2x\sqrt{x} = 0$

$16 = x\sqrt{x}$

$16 = x^{\frac{3}{2}}$

$16^2 = x^3$

$256 = x^3$

$4\sqrt[3]{4} = x$ $\quad (6.3496)$

$6.3496^2 + y^2 = 64\sqrt{6.3496} - y$

$y^2 - y = 120.9524$ $\quad y = 11.509\text{m}$ $\quad (-10.5$ not valid$)$

max height $= 11.5\text{m}$ (3SF)

13 (c) Suggest one limitation of the model Garry has used. **[1 mark]**

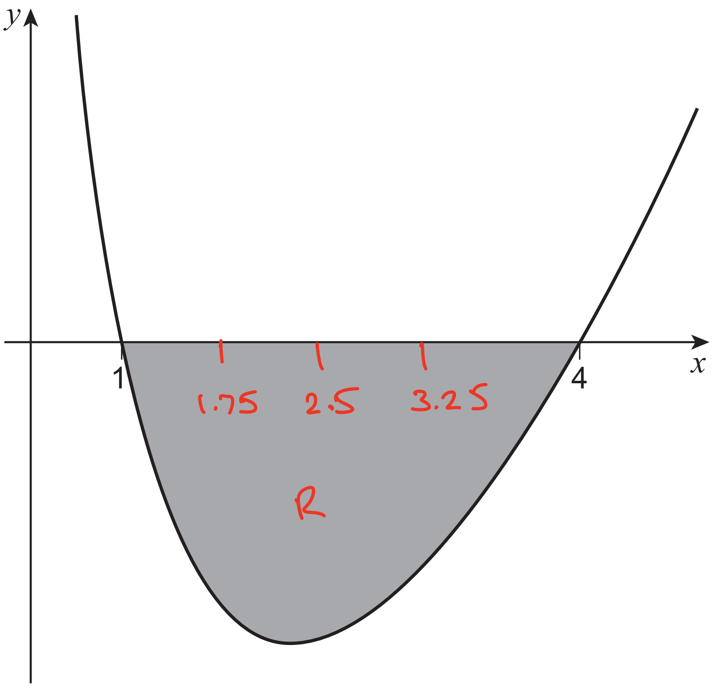
The entrance is unlikely to be perfectly smooth

Do not write outside the box

14 The region bounded by the curve

$$y = (2x - 8)\ln x$$

and the x-axis is shaded in the diagram below.



14 (a) Use the trapezium rule with **5 ordinates** to find an estimate for the area of the shaded region.

Give your answer correct to three significant figures.

[3 marks]

x =	1	1.75	2.5	3.25	4
$(2x-8)\ln x$:	0	−2.518	−2.749	−1.768	0

$h = 0.75$

$R = \frac{0.75}{2}\left\{0 + 2\left\{-2.518 - 2.749 - 1.768\right\}\right\}$

$= 5.27625$ 5.28 units^2 (3SF)

Do not write outside the box

14 (b) Show that the exact area is given by

$$32\ln 2 - \frac{33}{2}$$

Fully justify your answer.

[6 marks]

$\int_1^4 (2x-8)\ln x$ $\quad u = \ln x \quad \frac{dv}{dx} = 2x-8$

$\frac{du}{dx} = \frac{1}{x} \quad v = x^2 - 8x$

$= \left[\ln(x)(x^2-8x) - \int (x^2-8x)\frac{1}{x} \right]_1^4$

$= \left[(x^2-8x)(\ln x) - \int x-8 \right]_1^4$

$= \left[(x^2-8x)(\ln x) - \left[\frac{x^2}{2} - 8x \right] \right]^4$

$= \left[(x^2-8x)(\ln x) - \frac{x^2}{2} + 8x \right]_1^4$

$= \left[-16\ln 4 - 8 + 32 \right] - \left[-7 \times 0 - \frac{1}{2} + 8 \right]$

$= -16\ln 4 + 24 - \frac{15}{2} = -32\ln 2 + \frac{33}{2}$

As area can't be negative area $= 32\ln 2 - \frac{33}{2}$

Do not write outside the box

15 (a) Given that

$$y = \operatorname{cosec}\theta$$

15 (a) (i) Express y in terms of $\sin\theta$.

[1 mark]

$y = \frac{1}{\sin\theta}$

15 (a) (ii) Hence, prove that

$$\frac{dy}{d\theta} = -\operatorname{cosec}\theta\cot\theta$$

[3 marks]

$y = \frac{1}{\sin\theta}$ $\quad u = 1$ $\quad v = \sin\theta$

$\frac{du}{dx} = 0$ $\quad \frac{dv}{dx} = \cos\theta$

$\frac{dy}{dx} = \frac{0 - \cos\theta}{\sin^2\theta} = \frac{-\cos\theta}{\sin^2\theta} = \frac{-1}{\tan\theta \sin\theta}$

$= -\operatorname{cosec}\theta \cot\theta$

Do not write outside the box

15 (a) (iii) Show that

$$\frac{\sqrt{y^2 - 1}}{y} = \cos\theta \qquad \text{for } 0 < \theta < \frac{\pi}{2}$$

[3 marks]

$\frac{\sqrt{\operatorname{cosec}\theta^2 - 1}}{\operatorname{cosec}\theta} =$ $\qquad \operatorname{cosec}^2 = 1 + \cot^2$

$\frac{\sqrt{\cot^2\theta}}{\operatorname{cosec}\theta} = \frac{\cot\theta}{\operatorname{cosec}\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\operatorname{cosec}\theta}$

$= \frac{\cos\theta}{\cancel{\sin\theta}} \times \cancel{\sin\theta} = \cos\theta$

Do not write outside the box

15 (b) (i) Use the substitution

$$x = 2\,\text{cosec}\,u$$

to show that

$$\int \frac{1}{x^2\sqrt{x^2-4}}\,dx \qquad \text{for } x > 2$$

can be written as

$$k\int \sin u \; du$$

where k is a constant to be found.

[6 marks]

$\int \frac{1}{x^2\sqrt{x^2-4}}\,dx$

$x = 2\text{cosec}\,u$

$\frac{dx}{du} = -2\text{cosec}\,u\cot u$

$dx = -2\text{cosec}\,u\cot u\,du$

$\int \frac{1}{4\text{cosec}^2(u)\sqrt{4\text{cosec}^2 u - 4}}\,dx$

$\sqrt{4\text{cosec}^2 u - 4}$

$= 2\sqrt{\text{cosec}^2 u - 1}$

$= 2\sqrt{\cot^2 u} =$

$= 2\cot u$

$\int \frac{1}{4\text{cosec}^2 u} \times \frac{-2\text{cosec}\,u\cancel{\cot}\,u\,du}{2\cancel{\cot}\,u}$

$\int \frac{-1}{4\text{cosec}\,u}\,du = \int -\frac{\sin u\,du}{4} = -\frac{1}{4}\int \sin u\,du$

Do not write outside the box

15 (b) (ii) Hence, show

$$\int \frac{1}{x^2\sqrt{x^2-4}}\,\mathrm{d}x = \frac{\sqrt{x^2-4}}{4x} + c \qquad \text{for } x > 2$$

where c is a constant.

[3 marks]

$$-\frac{1}{4}\int \sin u \; du = -\frac{1}{4}\left[-\cos u\right]$$

$$= \frac{\cos u}{4} + c$$

$$x = 2\operatorname{cosec} u$$

$$x^2 = 4\operatorname{cosec}^2 u$$

$$x^2 = \frac{4}{\sin^2 u}$$

$$\sin^2 u = \frac{4}{x^2}$$

$$\cos u = \sqrt{1-\sin^2 u} = \sqrt{1-\frac{4}{x^2}}$$

$$\frac{\cos u}{4} = \frac{\sqrt{\frac{x^2-4}{x^2}}}{4} = \frac{\sqrt{x^2-4}}{4x} + c$$

END OF QUESTIONS