Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

A-level
MATHEMATICS
Paper 3

Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
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15	
16	
17	
18	
19	
TOTAL	

Section A

Answer **all** questions in the spaces provided.

1 State the range of values of x for which the binomial expansion of

$$\sqrt{1 - \frac{x}{4}}$$

is valid.

Circle your answer.

[1 mark]

$|x| < \frac{1}{4}$ $\qquad$ $|x| < 1$ $\qquad$ $|x| < 2$ $\qquad$ $|x| < 4$

$|x| < \frac{1}{\frac{1}{4}}$ $\qquad$ $|x| < 4$

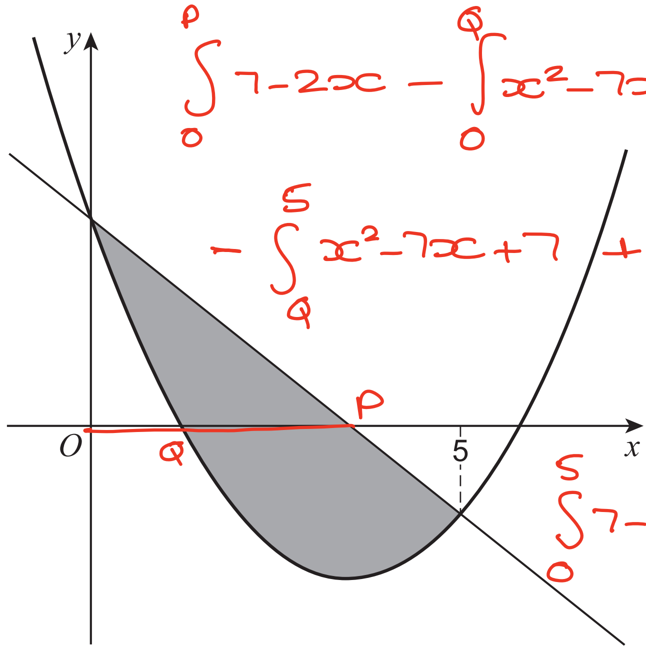
Do not write outside the box

2 The shaded region, shown in the diagram below, is defined by

$$x^2 - 7x + 7 \le y \le 7 - 2x$$

$\int_0^P 7-2x - \int_0^Q x^2-7x+7$

$-\int_Q^5 x^2-7x+7 + \int_P^5 7-2x$



$\int_0^5 7-2x - \int_0^5 x^2-7x+7$

Identify which of the following gives the area of the shaded region.

$= \int_0^5 -x^2+5x$

Tick (✓) **one** box.

[1 mark]

$\int (7 - 2x)\,dx - \int (x^2 - 7x + 7)\,dx$ ☐

$= \int_0^5 5x-x^2$

$\int_0^5 (x^2 - 5x)\,dx$ ☐

$\int_0^5 (5x - x^2)\,dx$ ☑

really tough question for 1 mark

$\int_0^5 (x^2 - 9x + 14)\,dx$ ☐

Turn over for the next question

Turn over ▶

Do not write outside the box

3 The function f is defined by

$y = 2x + 1$

$\frac{y-1}{2} = x$

$$\mathrm{f}(x) = 2x + 1$$

$\mathrm{f}^{-1}(x) = \frac{x-1}{2}$

Solve the equation

$$\mathrm{f}(x) = \mathrm{f}^{-1}(x)$$

Circle your answer.

[1 mark]

$x = -1$ $\quad$ $x = 0$ $\quad$ $x = 1$ $\quad$ $x = 2$

$2x + 1 = \frac{x-1}{2}$

$4x - 2 = x - 1$

$3x = -3 \quad x = -1$

4 Find

$$\int \left(x^2 + x^{\frac{1}{2}}\right) \mathrm{d}x$$

[2 marks]

$\frac{x^3}{3} + \frac{2}{3} x^{\frac{3}{2}} + C$

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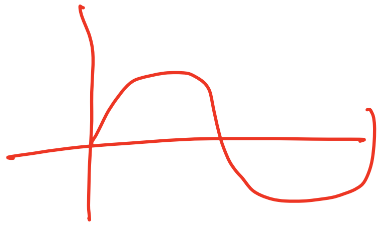
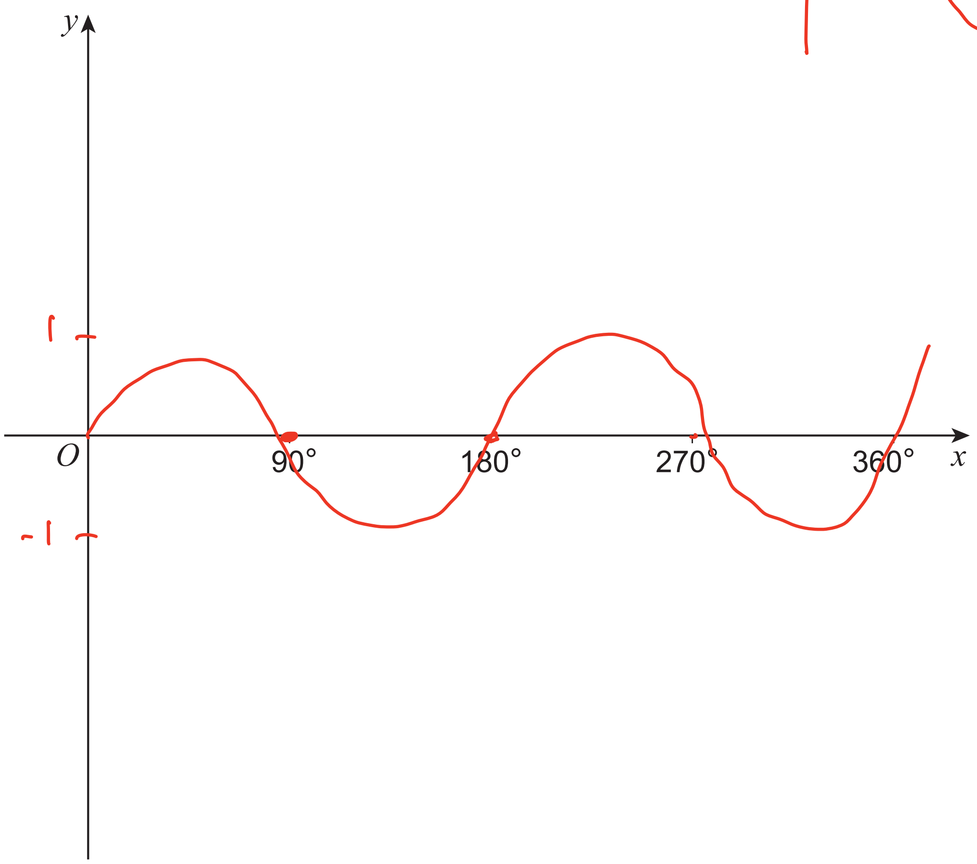
5 (a) Sketch the graph of

$$y = \sin 2x$$

for $0° \leq x \leq 360°$



sin 2



[2 marks]

5 (b) The equation

$$\sin 2x = A$$

has exactly two solutions for $0° \leq x \leq 360°$

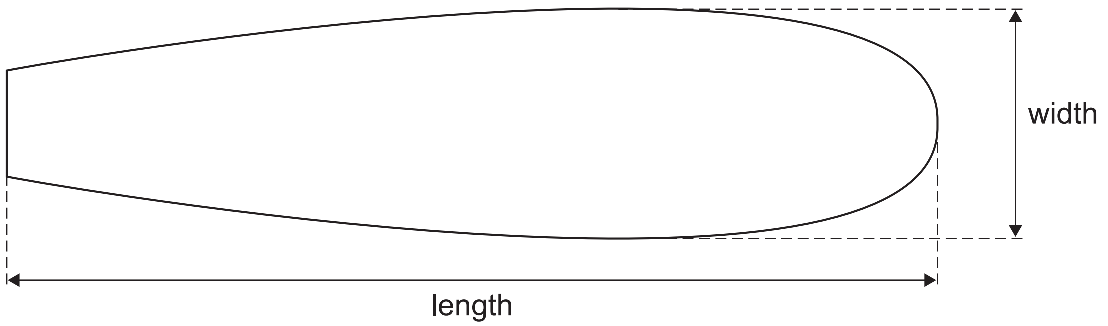
State the possible values of A.

[1 mark]

1 or -1

6 A design for a surfboard is shown in **Figure 1**.

Figure 1



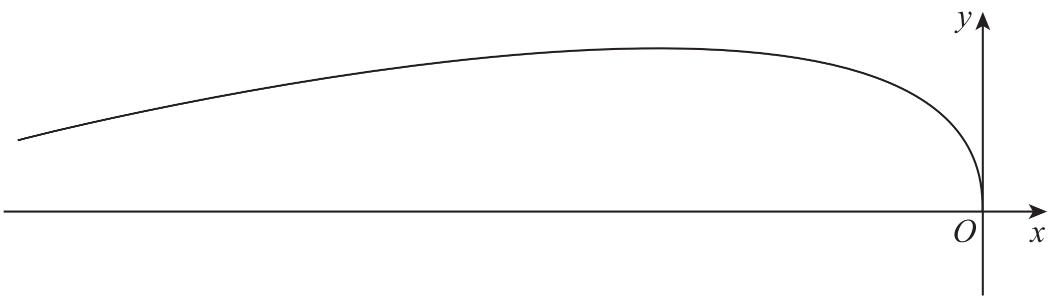
The curve of the **top half** of the surfboard can be modelled by the parametric equations

$$x = -2t^2$$

$$y = 9t - 0.7t^2$$

for $0 \leq t \leq 9.5$ as shown in **Figure 2**, where x and y are measured in centimetres.

Figure 2



6 (a) Find the length of the surfboard.

[2 marks]

$t=0 \quad x=0 \qquad t=9.5 \quad x = -2 \times 9.5^2 = -180.5$

length of the surfboard $= 180.5$ cm

Do not write outside the box

6 (b) (i) Find an expression for $\frac{dy}{dx}$ in terms of t.

$x = -2t^2$

[3 marks]

$y = 9t - 0.7t^2$

$\frac{dx}{dt} = -4t \qquad \frac{dy}{dt} = 9 - 1.4t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{9-1.4t}{-4t} = \frac{1.4t-9}{4t}$

6 (b) (ii) Hence, show that the width of the surfboard is approximately one third of its length.

[4 marks]

max height when $\frac{dy}{dx} = 0$

$\frac{1.4t-9}{4t} = 0 \qquad 1.4t - 9 = 0$

$1.4t = 9 \qquad t = \frac{9}{1.4}$

$y = 9\left(\frac{9}{1.4}\right) - 0.7\left(\frac{9}{1.4}\right)^2 = 28.93$

width $= 2 \times 28.93 = 57.857$

length was 180.5 cm width 57.9 cm

57.9 is approx $\frac{1}{3}$ of 180.5

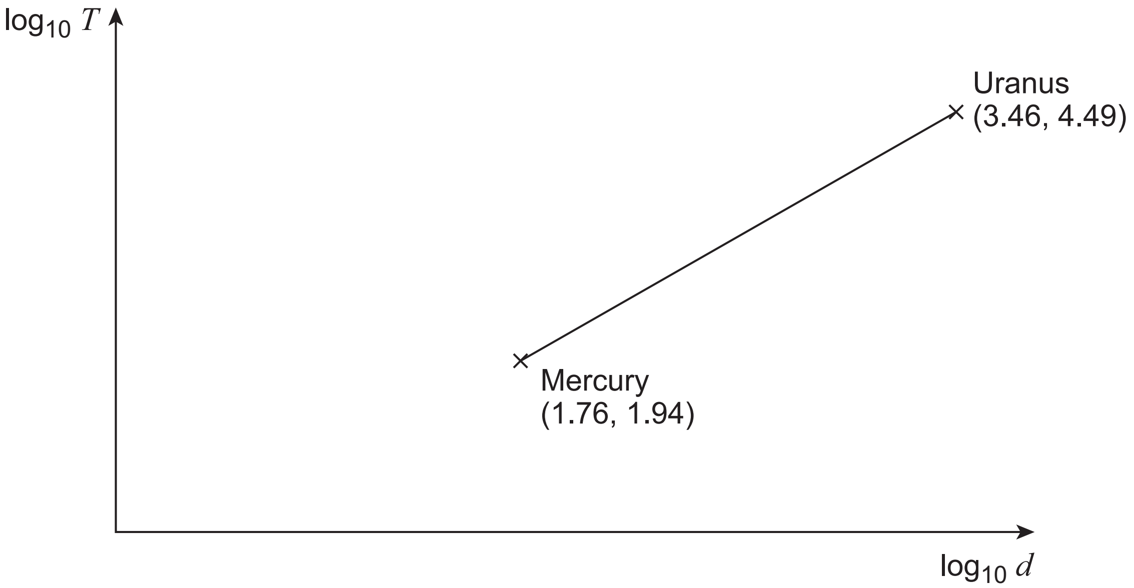
$180.5 \div 3 = 60.1$

7 A planet takes T days to complete one orbit of the Sun.

T is known to be related to the planet's average distance d, in millions of kilometres, from the Sun.

days   d millions km

A graph of $\log_{10} T$ against $\log_{10} d$ is shown with data for Mercury and Uranus labelled.



7 (a) (i) Find the equation of the straight line in the form

$$\log_{10} T = a + b \log_{10} d$$

where a and b are constants to be found.

[3 marks]

$\frac{\Delta y}{\Delta x} = \frac{2.55}{1.7} = 1.5$ $y = 1.5x + c$

$1.94 = 1.76(1.5) + c$

$-0.7 = c$

$\log_{10} T = -0.7 + 1.5 \log_{10} d$

Do not write outside the box

7 (a) (ii) Show that

$$T = \mathrm{K}\,d^{\mathrm{n}}$$

where K and n are constants to be found.

[2 marks]

0.7

$\log_{10} T = -0.7 + 1.5 \log_{10} d$

$T = 10^{-0.7} \times d^{1.5}$

$T = 0.2d^{1.5}$

7 (b) Neptune takes approximately 60 000 days to complete one orbit of the Sun.

Use your answer to **7(a)(ii)** to find an estimate for the average distance of Neptune from the Sun.

[2 marks]

$60000 = 0.2d^{1.5}$

$300{,}000 = d^{1.5}$

$d = 4481$

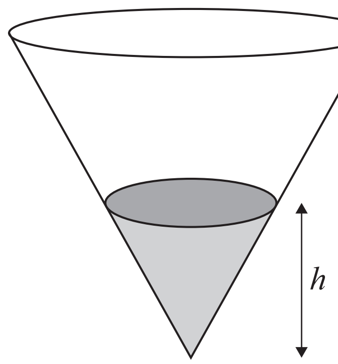
distance 4500 million km (2SF)

Turn over for the next question

Do not write outside the box

8 Water is poured into an empty cone at a constant rate of $8\text{ cm}^3/\text{s}$

After t seconds the depth of the water in the inverted cone is h cm, as shown in the diagram below.



When the depth of the water in the inverted cone is h cm, the volume, $V\text{ cm}^3$, is given by

$$V = \frac{\pi h^3}{12}$$

8 (a) Show that when $t = 3$

$$\frac{dV}{dh} = 6\sqrt[3]{6\pi}$$

[4 marks]

In $\frac{dv}{dt} = 8$ at t seconds $h = h$ $V = \frac{\pi h^3}{12}$ (1)

(2) $\frac{dv}{dh} = \frac{3\pi h^2}{12} = \frac{\pi h^2}{4}$

when $t = 3$

$V = 24$

(1) $24 = \frac{\pi h^3}{12}$

$288 = \pi h^3$

at $t = 3$ $\frac{288}{\pi} = h^3$

(2) $\frac{dv}{dh} = \frac{\pi}{4} \times \left(\frac{288}{\pi}\right)^{\frac{2}{3}} = \frac{\pi^{\frac{1}{3}}}{4} \sqrt[3]{82944}$

$\sqrt[3]{82944}$

6^3 384

2^3 48

$= 24\sqrt[3]{6}$

$= \frac{\pi^{\frac{1}{3}}}{4} \times 24\sqrt[3]{6}$

$6\sqrt[3]{6\pi}$

Do not write outside the box

8 (b) Hence, find the rate at which the depth is increasing when $t = 3$

Give your answer to three significant figures.

[3 marks]

$t=3$ $\quad \frac{dV}{dh} = 6\sqrt[3]{6\pi}$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $\quad$ Volume is increasing at $8cm^3/sec$

$\frac{dV}{dt} = 8$

$\frac{dh}{dt} = \frac{1}{6\sqrt[3]{6\pi}} \times 8 = \frac{4}{3\sqrt[3]{6\pi}} = 0.501 cms^{-1}$

Do not write outside the box

9 Assume that a and b are integers such that

$$a^2 - 4b - 2 = 0$$

9 (a) Prove that a is even.

[2 marks]

$a^2 = 4b + 2$

$a^2 = 2(2b+1)$ for a square number to be even its root must be even

9 (b) Hence, prove that $2b + 1$ is even and explain why this is a contradiction.

[3 marks]

if a is even it must be able to be written as $2n$

so $(2n)^2 = 4n^2$ if $4n^2 = 2(2b+1)$

$2n^2 = 2b+1$

$\therefore$ $\therefore$ $2b+1$ must be even

However 2 of anything + 1 is an odd number so this is a contradiction

Do not write outside the box

9 (c) Explain what can be deduced about the solutions of the equation

$$a^2 - 4b - 2 = 0$$

[1 mark]

because there is a contradiction, this equation must be unsolvedble where a and b are integers

Turn over for the next question

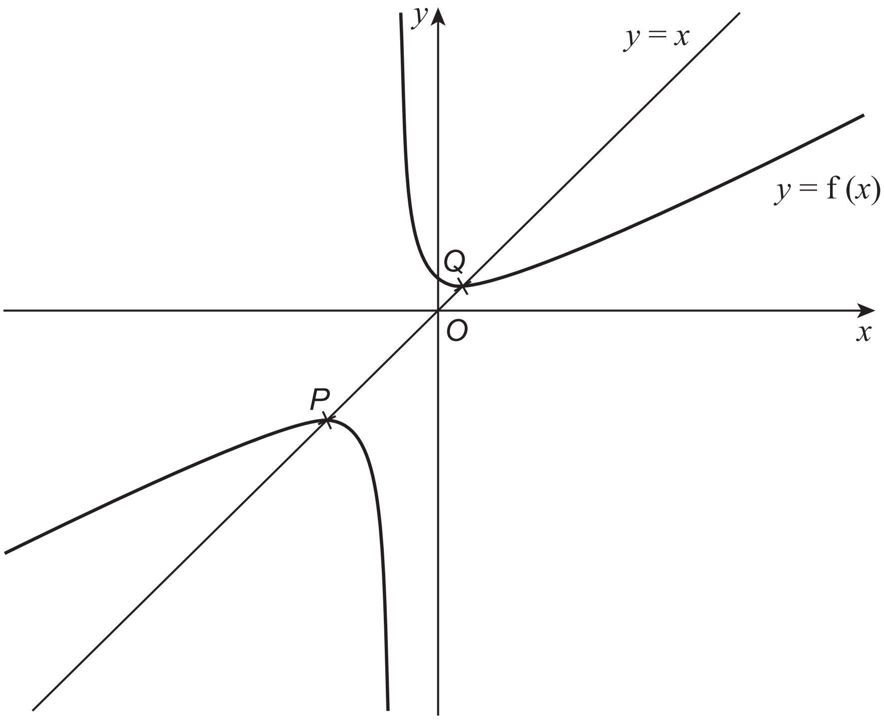
Do not write outside the box

10 The function f is defined by

$$\mathrm{f}(x) = \frac{x^2 + 10}{2x + 5}$$

where f has its maximum possible domain.

The curve $y = \mathrm{f}(x)$ intersects the line $y = x$ at the points P and Q as shown below.



10 (a) State the value of x which is not in the domain of f.

[1 mark]

$x = -\frac{5}{2}$ is not in the domain of f

Do not write outside the box

10 (b) Explain how you know that the function f is many-to-one.

[2 marks]

Because 2 or more values of x give the same value of y

10 (c) (i) Show that the x-coordinates of P and Q satisfy the equation

$$f(x) = \frac{x^2 + 10}{2x + 5} \qquad x^2 + 5x - 10 = 0$$

[2 marks]

when $x =$

10 (c) (ii) Hence, find the exact x-coordinate of P and the exact x-coordinate of Q.

[1 mark]

$\left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{40}{4} = 0$

$x + \frac{5}{2} = \pm \frac{\sqrt{65}}{2}$

$x = \frac{-5 \pm \sqrt{65}}{2}$

P is most left so the x coordinate of $P = \frac{-5 - \sqrt{65}}{2}$

$Q = \frac{-5 + \sqrt{65}}{2}$

Turn over ▶

Do not write outside the box

10 (d) Show that P and Q are stationary points of the curve.

Fully justify your answer.

[5 marks]

$$f(x) = \frac{x^2 + 10}{2x + 5}$$

let $u = x^2 + 10$ $\quad v = 2x + 5$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dx} = 2$

$f'(x) = \frac{(2x+5)(2x) - [2(x^2+10)]}{(2x+5)^2}$

$= \frac{4x^2 + 10x - 2x^2 - 20}{(2x+5)^2} = \frac{2x^2 + 10x - 20}{(2x+5)^2}$

Turning points when $f'(x) = 0$

$2x^2 + 10x - 20 = 0$

$x^2 + 5x - 10 = 0$ $\quad$ i.e. $x = \frac{-5 \pm \sqrt{65}}{2}$ (as before)

so P and Q are stationary points

10 (e) Using set notation, state the range of f.

[2 marks]

$$f(x) = \frac{x^2 + 10}{2x + 5}$$

when $x = \frac{-5 + \sqrt{65}}{2}$

$f(x) = \frac{\left(\frac{-5+\sqrt{65}}{2}\right)^2 + 10}{-5 + \sqrt{65} + 5} = \frac{\frac{25 - 10\sqrt{65} + 65 + 40}{4}}{\sqrt{65}}$

$f(x) = \frac{130 - 10\sqrt{65}}{4\sqrt{65}} = \frac{65 - 5\sqrt{65}}{2\sqrt{65}} \times \frac{\sqrt{65}}{\sqrt{65}}$

$= \frac{65\sqrt{65} - 325}{130}$

$= \frac{\sqrt{65} - 5}{2}$

when $x = \frac{-5 - \sqrt{65}}{2}$ $= \frac{\frac{25 + 10\sqrt{65} + 65 + 40}{4}}{-5 - \sqrt{65} + 5}$

$= \frac{130 + 10\sqrt{65}}{-4\sqrt{65}} = \frac{65 + 5\sqrt{65}}{-2\sqrt{65}}$

$= \frac{\sqrt{65} + 5}{} = -5 - \sqrt{65}$ $\quad =$

$\left\{f(x): f(x) \leq \frac{-5-\sqrt{65}}{2}\right\} \cup \left\{f(x): f(x) \geq \frac{-5+\sqrt{65}}{2}\right\}$

Section B

Answer **all** questions in the spaces provided.

11 $X \sim \mathrm{N}(14, 0.35)$

Find the standard deviation of X, correct to two decimal places.

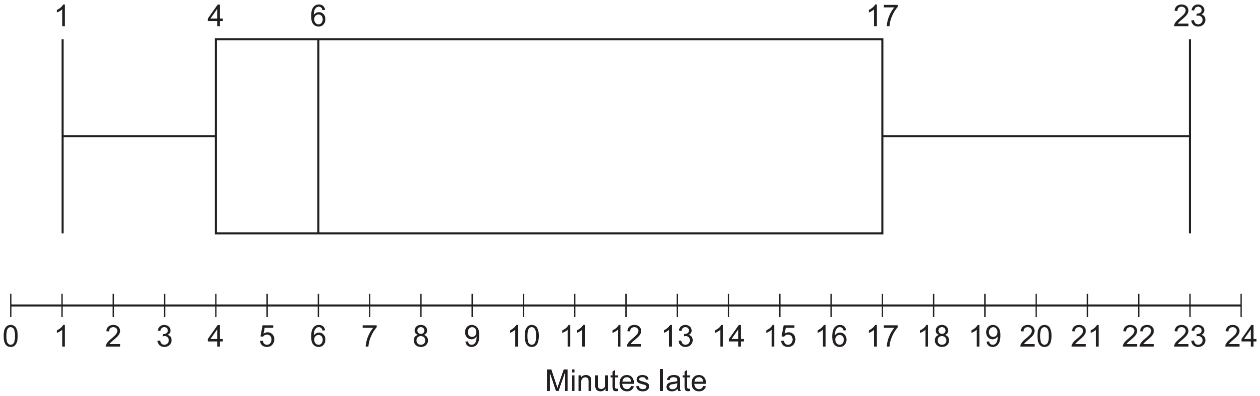
Circle your answer.

[1 mark]

0.12 0.35 0.59 1.78

$\sqrt{0.35} = 0.5916$

12 The box plot below shows summary data for the number of minutes late that buses arrived at a rural bus stop.



Identify which term best describes the distribution of this data.

Circle your answer. left skew = positively skewed

[1 mark]

negatively skewed normal positively skewed symmetrical

Turn over for the next question

13 A reporter is writing an article on the CO_2 emissions from vehicles using the Large Data Set.

The reporter claims that the Large Data Set shows that the CO_2 emissions from all vehicles in the UK have declined every year from 2002 to 2016.

Using your knowledge of the Large Data Set, give **two** reasons why this claim is invalid.

[2 marks]

Not all UK regions are included so can't conclude anything about all vehicles

Not all types of vehicles are included either

Do not write outside the box

14 A customer service centre records every call they receive.

It is found that 30% of all calls made to this centre are complaints.

A sample of 20 calls is selected.

The number of calls in the sample which are complaints is denoted by the random variable X.

14 (a) State **two** assumptions necessary for X to be modelled by a binomial distribution.

[2 marks]

- Each call is independent of every other
- The probability of each call being a complaint remains constant

14 (b) Assume that X can be modelled by a binomial distribution.

14 (b) (i) Find $\mathrm{P}(X = 1)$

[1 mark]

$X \sim B(20, 0.3)$

$P(X=1) = 0.0068$

14 (b) (ii) Find $\mathrm{P}(X < 4)$ 0, 1, 2, 3

[2 marks]

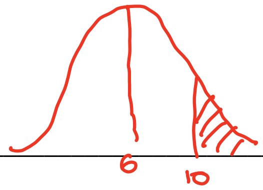
$P(X<4) = P(X \leq 3)$

$= 0.1071$ (4dp)

Do not write outside the box

14 (b) (iii) Find $\mathrm{P}(X \geq 10)$

[2 marks]



$Px \geq 10 = 1 - P(x \leq 9)$

$= 1 - 0.952038\ldots$

$= 0.04796$

14 (c) In a random sample of 10 calls to a school, the number of calls which are complaints, Y, may be modelled by a binomial distribution

$$Y \sim \mathrm{B}(10, p)$$

$np, \ np(1-p)$

The standard deviation of Y is 1.5

Calculate the possible values of p.

[3 marks]

$np(1-p) = 1.5^2$

$np - np^2 = 2.25$

$10p - 10p^2 = 2.25$

$0 = 10p^2 - 10p + 2.25$

$p = 0.6581$

$p = 0.3419$

Turn over for the next question

15 Researchers are investigating the average time spent on social media by adults on the electoral register of a town.

They select every 100th adult from the electoral register for their investigation.

15 (a) Identify the population in their investigation.

[1 mark]

Adults from the electoral register

15 (b) (i) State the name of this method of sampling.

[1 mark]

Systematic sampling

15 (b) (ii) Describe **one** advantage of this sampling method.

[1 mark]

Its convenient and cheap to collect

16 A sample of 240 households were asked which, if any, of the following animals they own as pets:

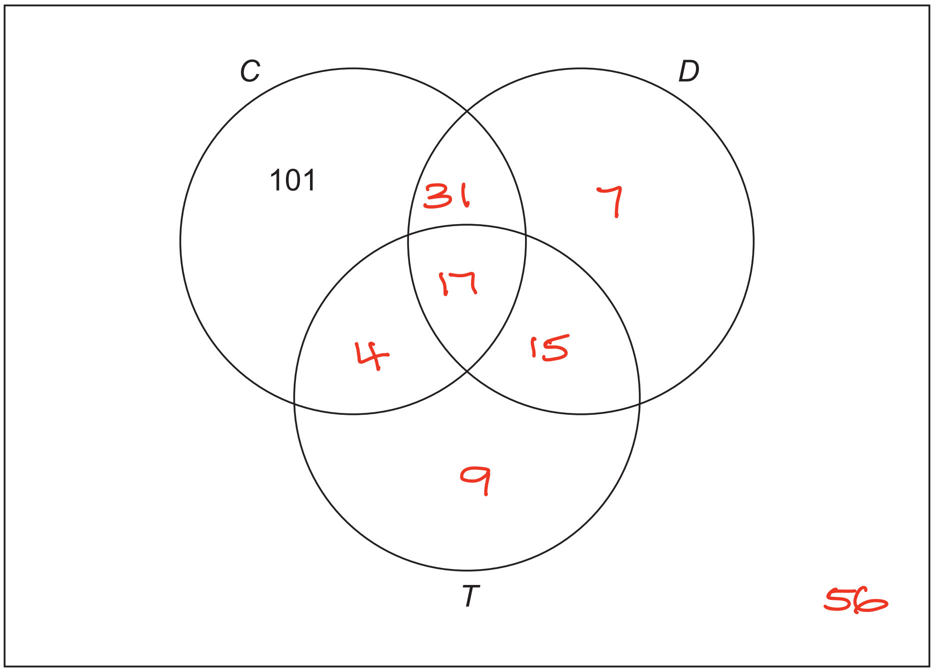
- cats (C)
- dogs (D)
- tortoises (T)

The results are shown in the table below.

Types of pet	C	D	T	C and D	C and T	D and T	C, D and T
Number of households	153	70	45	48	21	32	17

16 (a) Represent this information by fully completing the Venn diagram below.

[3 marks]



153 +
22
9
184

16 (b) A household is chosen at random from the sample.

16 (b) (i) Find the probability that the household owns a cat only.

[1 mark]

P(cat only) $= \dfrac{101}{240}$

16 (b) (ii) Find the probability that the household owns at least two of the three types of pet.

[2 marks]

P(at least 2) = $\frac{31 + 17 + 4 + 15}{240}$ = $\frac{67}{240}$

16 (b) (iii) Find the probability that the household owns a cat or a dog or both, given that the household does not own a tortoise.

[2 marks]

number not own tortoise = 195

no cat dog or both / not tortoise = 139

P(cat dog or both / no tortoise) = $\frac{139}{195}$

16 (c) Determine whether a household owning a cat and a household owning a tortoise are independent of each other.

Fully justify your answer.

[2 marks]

P(Cat) = $\frac{153}{240}$ P(tortoise) = $\frac{45}{240}$

If independent $P(C \cap T) = \frac{153}{240} \times \frac{45}{240} = \frac{153}{1280}$ (0.1195…)

when actual $P(C \cap T)$ $\frac{21}{240}$ (0.0875…)

so they are not independent of each other

Do not write outside the box

17 The number of working hours per week of employees in a company is modelled by a normal distribution with mean of 34 hours and a standard deviation of 4.5 hours.

The manager claims that the mean working hours per week of the company's employees has increased.

A random sample of 30 employees in the company was found to have mean working hours per week of 36.2 hours.

Carry out a hypothesis test at the 2.5% significance level to investigate the manager's claim.

[6 marks]

$X \sim N(34, 4.5^2)$

X = no of working hours per week of employees

$H_0: \mu = 34$

$H_1: \mu > 34$

sample 30

one tailed so significance level 0.025

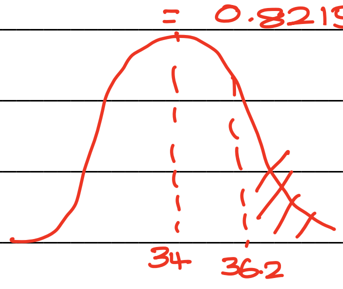
Sample mean 36.2

σ sample $= \sqrt{\dfrac{4.5^2}{30}}$

$= 0.82158$

$X \sim N(34, 0.82158^2)$

$P(X > 36.2) = 0.00371$



as $0.00371 < 0.025$

This is significant enough to reject H_0 in favour of H_1 and agree with the manager's assertions that mean working hours has increased

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18 In a particular year, the height of a male athlete at the Summer Olympics has a mean 1.78 metres and standard deviation 0.23 metres.

The heights of 95% of male athletes are between 1.33 metres and 2.22 metres.

18 (a) Comment on whether a normal distribution may be suitable to model the height of a male athlete at the Summer Olympics in this particular year.

[3 marks]

$\sigma = 0.23$ For it 2 be a normal distribution 95% of the population should be $\pm 2\sigma$ from the mean and its must be continuous data

$1.78 + 2(0.23) = 2.24$

$1.78 - 2(0.23) = 1.32$ so as this is reasonably close a normal distribution may be suitable

18 (b) You may assume that the height of a male athlete at the Summer Olympics may be modelled by a normal distribution with mean 1.78 metres and standard deviation 0.23 metres.

18 (b) (i) Find the probability that the height of a randomly selected male athlete is 1.82 metres.

[1 mark]

P(exactly 1.82 metres) = 0

18 (b) (ii) Find the probability that the height of a randomly selected male athlete is between 1.70 metres and 1.90 metres.

[1 mark]

$P(1.70 < X < 1.90) = 0.3350886\ldots$

$= 0.3351$

Do not write outside the box

18 (b) (iii) Two male athletes are chosen at random.

Calculate the probability that **both** of their heights are between 1.70 metres and 1.90 metres.

[1 mark]

$(0.3351)^2 = 0.1122843....$

$= 0.1123$ (4dp)

18 (c) The summarised data for the heights, h metres, of a random sample of 40 male athletes at the Winter Olympics is given below.

$$\sum h = 69.2 \qquad \sum (h - \bar{h})^2 = 2.81$$

Use this data to calculate estimates of the mean and standard deviation of the heights of male athletes at the Winter Olympics.

[3 marks]

$\sqrt{\frac{2.81}{40}} = 0.2650$ (4dp)

18 (d) Using your answers from **part (c)**, compare the heights of male athletes at the Summer Olympics and male athletes at the Winter Olympics.

[2 marks]

winter olympics $\sigma = 0.265$ $\mu = 1.73$ $\frac{69.2}{40}$

summer olympics $\sigma = 0.23$ $\mu = 1.78$

Summer olympics atheletes tend to be taller and have a a smaller spread of heights

19 A bank runs a campaign to promote Internet banking accounts to their customers.

Before the campaign, 42% of their customers had an Internet banking account.

One week after the campaign started, 35 customers were surveyed at random and 18 of them were found to have registered for an Internet banking account.

Using a binomial distribution, carry out a hypothesis test at the 10% significance level to investigate the claim that, since the campaign, there has been an increase in the proportion of customers registered for an Internet banking account.

[6 marks]

$X \sim B(35, 0.42)$

$H_0: p = 0.42$

$H_1: p > 0.42$

X = no of people in a sample of 35 who have an internet banking account

$P(X \geq 18)$

$= 1 - P(X \leq 17)$

$= 1 - 0.8313615\ldots$

$= 0.1686$

$0.1686 > 0.10$

So there is insufficient evidence to reject H_0 in favour of H_1. There is insufficient evidence to suggest that there has been an increase in the number of internet banking customers

END OF QUESTIONS

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