Please write clearly in block capitals.

Centre number						Candidate number				

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

A-level
MATHEMATICS
Paper 2

Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
TOTAL	

Do not write outside the box

Section A

Answer **all** questions in the spaces provided.

1 A circle has centre (4, -5) and radius 6

Find the equation of the circle.

Tick (✓) **one** box.

[1 mark]

- [ ] $(x-4)^2+(y+5)^2=6$
- [ ] $(x+4)^2+(y-5)^2=6$
- [x] $(x-4)^2+(y+5)^2=36$
- [ ] $(x+4)^2+(y-5)^2=36$

2 State the value of

$$\lim_{h\to 0}\frac{\sin(\pi+h)-\sin\pi}{h}$$

Circle your answer.

[1 mark]

$\cos h$ -1 0   1

$\cos\pi = -1$

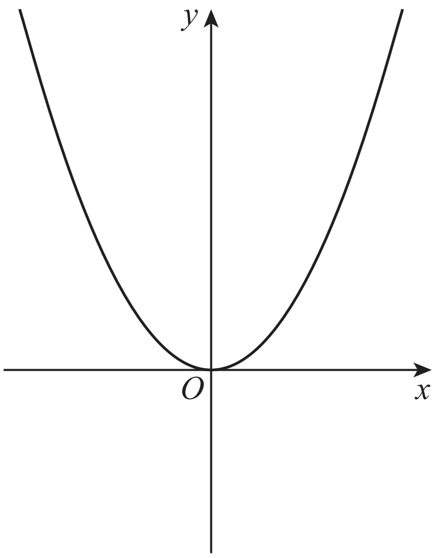
Do not write outside the box

3 The function f is concave and is represented by one of the graphs below.

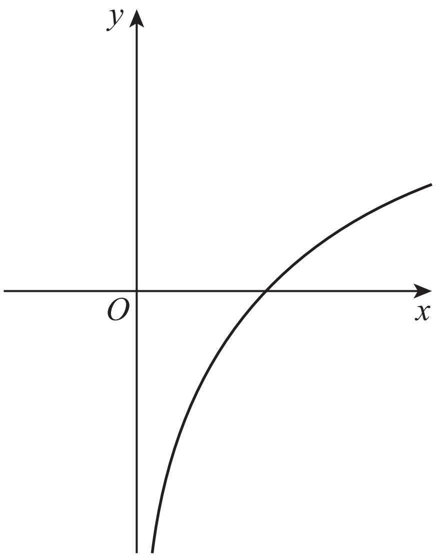
Identify the graph which represents f.

Tick (✓) **one** box.

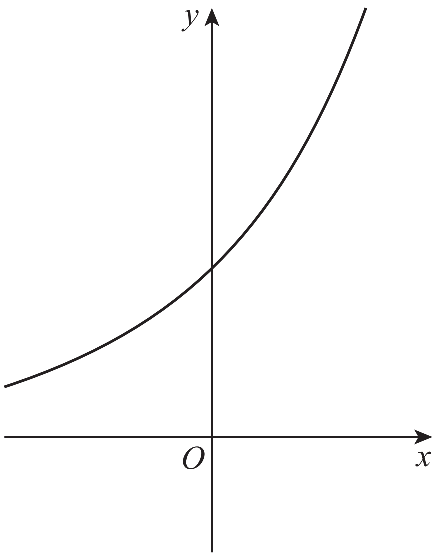
[1 mark]



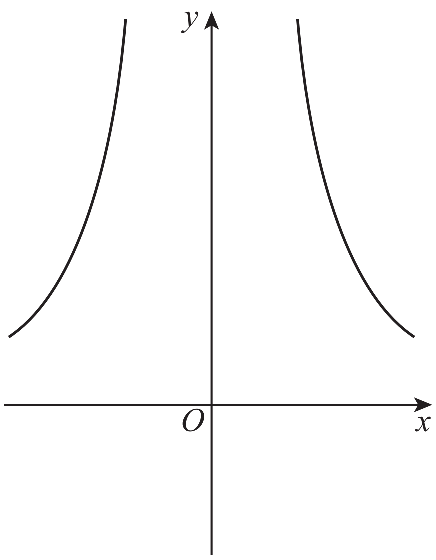
☐



☑



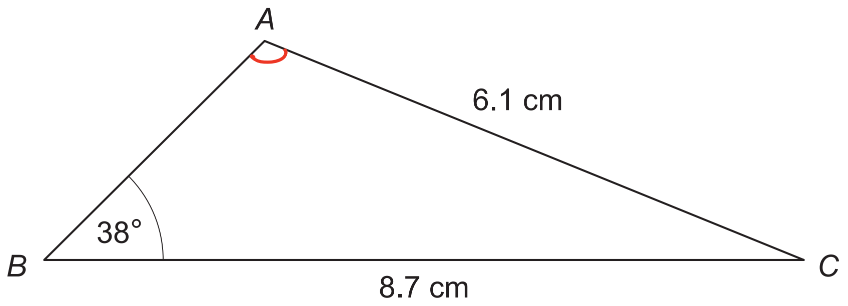
☐



☐

Do not write outside the box

4



The diagram shows a triangle *ABC*.

AB is the shortest side. The lengths of *AC* and *BC* are 6.1 cm and 8.7 cm respectively.

The size of angle *ABC* is 38°

Find the size of the largest angle.

Give your answer to the nearest degree.

[3 marks]

$\frac{6.1}{\sin 38} = \frac{8.7}{\sin x}$ $x = \sin^{-1} \frac{8.7}{6.1} \times \sin 38$

$x = 61.4,\ 180 - 61.4°$

as x is obtuse

BAC = 119° nearest degree

Do not write outside the box

5 The binomial expansion of $(2+5x)^4$ is given by

$$(2+5x)^4 = A + 160x + Bx^2 + 1000x^3 + 625x^4$$

5 (a) Find the value of A and the value of B.

[2 marks]

$(2+5x)^4 = 2^4 + 2^3(4)(5)x + 2^2\frac{(4)(3)}{2}(5^2)x^2 + \ldots$

$= 16 + 160x + 600x^2 +$

$A = 16$

$B = 600$

5 (b) Show that

$$(2+5x)^4 - (2-5x)^4 = Cx + Dx^3$$

where C and D are constants to be found.

[2 marks]

$(2-5x)^4 = 16 - 160x + 600x^2 - 1000x^3 + 625x^4$

$(2+5x)^4 = 16 + 160x + 600x^2 + 1000x^3 + 625x^4$

$(2+5x)^4 - (2-5x)^4 = 320x + 2000x^3$

Do not write outside the box

5 (c) Hence, or otherwise, find

$$\int \left((2+5x)^4 - (2-5x)^4 \right) \mathrm{d}x$$

[2 marks]

$\int 320x + 2000x^3$

$= 160x^2 + 500x^4 + C$

Turn over for the next question

Do not write outside the box

6 (a) Asif notices that $24^2 = 576$ and $2 + 4 = 6$ gives the last digit of 576

He checks two more examples:

$27^2 = 729$	$29^2 = 841$
$2 + 7 = 9$	$2 + 9 = 11$
Last digit 9	Last digit 1

Asif concludes that he can find the last digit of any square number greater than 100 by adding the digits of the number being squared.

Give a counter example to show that Asif's conclusion is **not** correct.

[2 marks]

$31^2 = 961$ but $3+1 = 4$ not 1

6 (b) Claire tells Asif that he should look only at the last digit of the number being squared.

$27^2 = 729$	$24^2 = 576$
$7^2 = 49$	$4^2 = 16$
Last digit 9	Last digit 6

Using Claire's method determine the last digit of 23456789^2

[1 mark]

$9^2 = 81$ so last digit $= 1$

Do not write outside the box

6 (c) Given Claire's method is correct, use proof by exhaustion to show that no square number has a last digit of 8

[2 marks]

$1^2 = 1$ as each subsequent square

$2^2 = 4$ number will from Claire's method

$3^2 = 9$ either end in 1 4 9 6 or 5

$4^2 = 16$ no square number ends in a 9

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

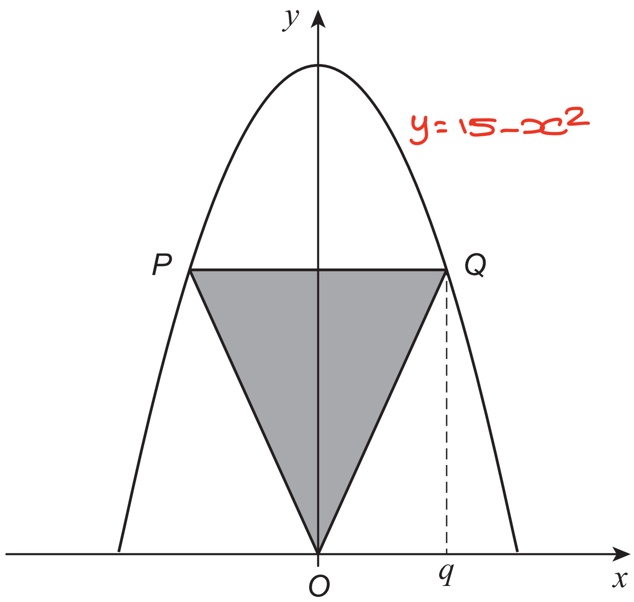
$9^2 = 81$

$10^2 = 100$

Turn over for the next question

Do not write outside the box

7 The curve $y = 15 - x^2$ and the isosceles triangle OPQ are shown on the diagram below.



Vertices P and Q lie on the curve such that Q lies vertically above some point $(q, 0)$

The line PQ is parallel to the x-axis.

7 (a) Show that the area, A, of the triangle OPQ is given by

$$A = 15q - q^3 \quad \text{for } 0 < q < c$$

where c is a constant to be found.

[3 marks]

height Q = $(15 - q^2)$

base triangle = $2q$

Area = $\frac{1}{2} \times 2q \times (15 - q^2) = q(15 - q^2)$

$= 15q - q^3$

$y = 15 - x^2$ when $y = 0$ $x^2 = 15$ $x = \pm\sqrt{15}$

$c = \sqrt{15}$

Do not write outside the box

7 (b) Find the exact maximum area of triangle *OPQ*.

Fully justify your answer.

[6 marks]

Area = $15q - q^3$

max area when $\frac{DA}{dq} = 0$ $\quad 15 - 3q^2 = 0$

$q^2 = 5$

$q = \sqrt{5}$

$\frac{D^2A}{dq^2} = -6q$ when $q = \sqrt{5}$ $\frac{D^2A}{dq^2} < 0$ so a max value

when $q = \sqrt{5}$ $\quad 15q - q^3$

max area = $15\sqrt{5} - 5\sqrt{5} = 10\sqrt{5}$ unit^2

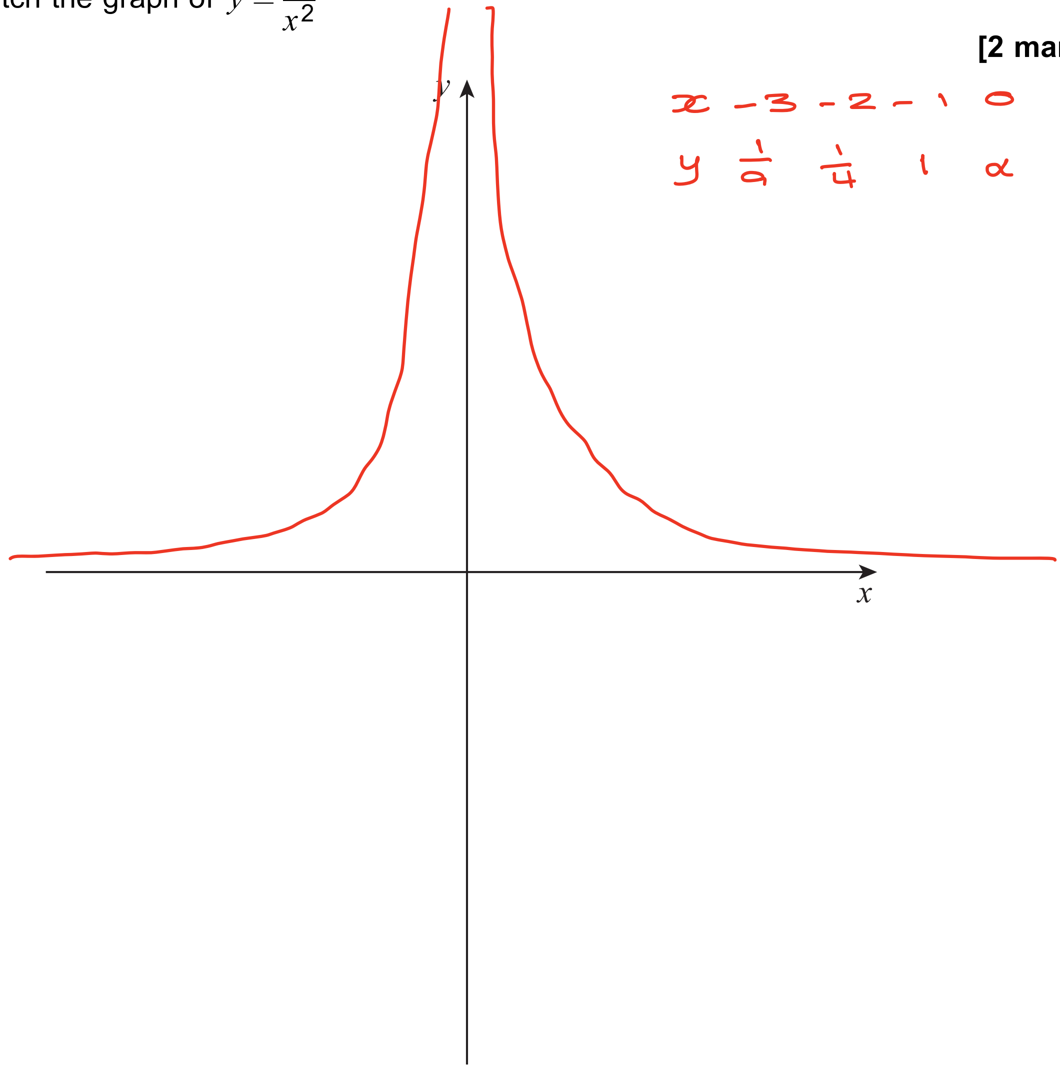
Turn over for the next question

Do not write outside the box

8 (a) Sketch the graph of $y = \dfrac{1}{x^2}$

[2 marks]

x	-3	-2	-1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{4}$	1	α	1	$\frac{1}{4}$



Do not write outside the box

8 (b) The graph of $y = \frac{1}{x^2}$ can be transformed onto the graph of $y = \frac{9}{x^2}$ using a stretch in one direction.

Beth thinks the stretch should be in the y-direction.

Paul thinks the stretch should be in the x-direction.

State, giving reasons for your answer, whether Beth is correct, Paul is correct, both are correct or neither is correct.

[3 marks]

Beth is correct as it is a stretch in the y direction scale factor 9

However Paul is also correct as x could be substituted with $\frac{x}{3}$ to make $y = \frac{9}{x^2}$ so it could also be described as a stretch in the x direction scale factor 3

So both are correct

Turn over for the next question

Do not write outside the box

9 Given that

$$\log_2 x^3 - \log_2 y^2 = 9$$

show that

$$x = Ay^p$$

where A is an integer and p is a rational number.

[4 marks]

$\log_2 \frac{x^3}{y^2} = 9$

$\frac{x^3}{y^2} = 2^9$

$x^3 = 512y^2$

$x = \sqrt[3]{512y^2}$

$x = 8y^{\frac{2}{3}}$

Do not write outside the box

10 A gardener has a greenhouse containing 900 tomato plants.

The gardener notices that some of the tomato plants are damaged by insects.

Initially there are 25 damaged tomato plants.

The number of tomato plants damaged by insects is increasing by 32% each day.

10 (a) The total number of plants damaged by insects, x, is modelled by

$$x = A \times B^t$$

where A and B are constants and t is the number of days after the gardener first noticed the damaged plants.

10 (a) (i) Use this model to find the total number of plants damaged by insects 5 days after the gardener noticed the damaged plants.

[3 marks]

$25,\ 25 \times 1.32,\ 25 \times 1.32^2$

$x = 25 \times 1.32^t$ $\quad 25 \times 1.32^5 = 100.1866$

101 plants were damaged

10 (a) (ii) Explain why this model is not realistic in the long term.

[2 marks]

There are not an infinite number of plants but this model has no upper limit

Do not write outside the box

10 (b) A refined model assumes the rate of increase of the number of plants damaged by insects is given by

$$\frac{dx}{dt} = \frac{x(900 - x)}{2700}$$

10 (b) (i) Show that

$$\int \left(\frac{A}{x} + \frac{B}{900 - x} \right) dx = \int dt$$

where A and B are positive integers to be found.

[3 marks]

$\int \frac{2700}{x(900-x)} = \int dt$

$\frac{2700}{x(900-x)} = \frac{A}{x} + \frac{B}{900-x}$

$2700 = (900-x)A + xB$

let $x=0$ $2700 = 900A$ let $x = 900$

$A=3$ $2700 = 900B$ $B=3$

$\int \frac{3}{x} + \frac{3}{900-x} = \int dt$

Question 10 continues on the next page

Do not write outside the box

10 (b) (ii) Hence, find t in terms of x.

[5 marks]

$$\int \frac{3}{x} + \frac{3}{900-x}\,dx = \int dt$$

$= 25$

$t = 0 \quad x = 25$

$3\ln x - 3\ln(900-x) = t + c$

$3\ln 25 - 3\ln(875) = c$

$3\ln \frac{25}{875} = c$

$3\ln \frac{1}{35} = c \qquad c = \ln\left(\frac{1}{35}\right)^3$

$3\ln x - 3\ln(900-x) = t + 3\ln\frac{1}{35}$

$3\ln x - 3\ln(900-x) - 3\ln\frac{1}{35} = t$

$3\ln\left(\frac{x}{900-x} \times 35\right) = t$

$3\ln\left(\frac{35x}{900-x}\right) = t$

10 (b) (iii) Hence, find the number of days it takes from when the damage is first noticed until half of the plants are damaged by the insects.

[2 marks]

half of 900 plants $= 450$ plants

$3\ln\left(\frac{35 \times 450}{450}\right) = t$

$3\ln 35 = t = 10.666 \qquad$ 11days

Do not write outside the box

Section B

Answer **all** questions in the spaces provided.

11 A moon vehicle has a mass of 212 kg and a length of 3 metres.

On the moon the vehicle has a weight of 345 N

Calculate a value for acceleration due to gravity on the moon.

Circle your answer.

[1 mark]

$0.614\ \text{m s}^{-2}$ $1.63\ \text{m s}^{-2}$ $1.84\ \text{m s}^{-2}$ $4.89\ \text{m s}^{-2}$

345 = 212a
= a

12 A car is travelling along a straight horizontal road with initial velocity $u\ \text{m s}^{-1}$

The car begins to accelerate at a constant rate $a\ \text{m s}^{-2}$ for 5 seconds, to reach a final velocity of $4u\ \text{m s}^{-1}$

Express a in terms of u.

Circle your answer.

[1 mark]

$a = 0.2u$ $a = 0.4u$ $a = 0.6u$ $a = 0.8u$

u = u
a = a
t = 5
v = 4u

v = u + at
4u = u + 5a
3u = 5a
$\frac{3}{5}u = a$

Turn over for the next question

Do not write outside the box

13 **In this question use $g = 9.8\,\text{m}\,\text{s}^{-2}$**

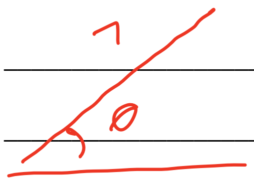
A ball is projected from a point on horizontal ground with an initial velocity of $7\,\text{m}\,\text{s}^{-1}$ at an angle θ above the horizontal.

The ball reaches a maximum vertical height of h metres above the ground.

13 (a) Show that

$$h = 2.5\sin^2\theta$$

[3 marks]



$s = h$
$u = 7\sin\theta$
$v = 0$
$a = -9.8$
t

$v^2 = u^2 + 2as$

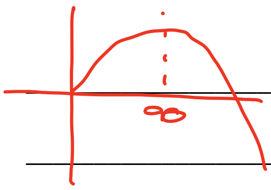
$0 = 49\sin^2\theta - 19.6h$

$19.6h = 49\sin^2\theta$

$h = 2.5\sin^2\theta$

13 (b) Hence, given that $0° \leq \theta \leq 60°$, find the maximum value of h.

[2 marks]



max value $\sin\theta$ between $0 \leq \theta \leq 60$

$= \sin 60 = \frac{\sqrt{3}}{2}$

So max height $= 2.5\sin^2\theta = 2.5 \times \frac{3}{4} = \frac{7.5}{4}$

$= 1.875\text{m}$

Do not write outside the box

13 (c) Nisha claims that the larger the size of the ball, the greater the maximum vertical height will be.

State whether Nisha is correct, giving a reason for your answer.

[1 mark]

downward force is increased as mass increases so no the height will not increase as max increases

In addition to this wind resistance will be greater

Turn over for the next question

Do not write outside the box

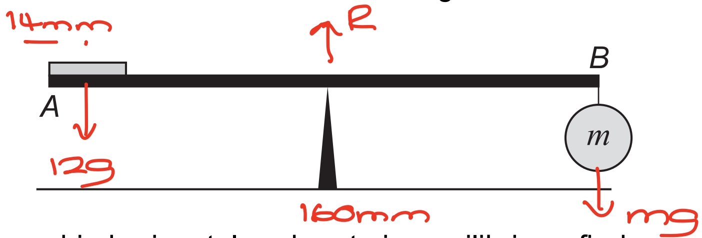
14 A £2 coin has a diameter of 28 mm and a mass of 12 grams.

A uniform rod AB of length 160 mm and a fixed load of mass m grams are used to check that a £2 coin has the correct mass.

The rod rests with its midpoint on a support.

A £2 coin is placed face down on the rod with part of its curved edge directly above A.

The fixed load is hung by a light inextensible string from a point directly below the other end of the rod at B, as shown in the diagram.



14 (a) Given that the rod is horizontal and rests in equilibrium, find m.

[3 marks]

moments about centre

$12g \times 66 = mg \times 80$

$9.9 = m$

14 (b) State an assumption you have made about the £2 coin to answer **part (a)**.

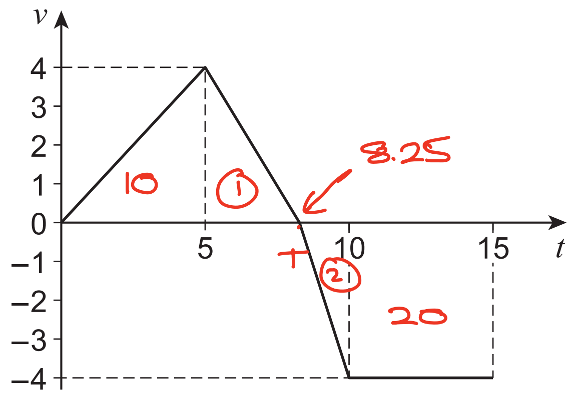
[1 mark]

I have assumed the coin is uniform

Do not write outside the box

15 A car is moving in a straight line along a horizontal road.

The graph below shows how the car's velocity v m s^{-1} changes with time, t seconds.



Over the period $0 \leq t \leq 15$ the car has a total displacement of -7 metres.

Initially the car has velocity 0 m s^{-1}

Find the next time when the velocity of the car is 0 m s^{-1}

[4 marks]

Area 1 $= (T-5)\times 2 = 2T-10$

2 $= (10-T)\times 2 \quad 20-2T$

$10 + 2T - 10 - (20-2T) - 20 = -7$

$4T - 40 = -7$

$4T = 33 \quad T = \frac{33}{4} = 8.25$ sec

Do not write outside the box

16 Two particles, P and Q, move in the same horizontal plane.

Particle P is initially at rest at the point with position vector $(-4\mathbf{i} + 5\mathbf{j})$ metres and moves with constant acceleration $(3\mathbf{i} - 4\mathbf{j})\,\mathrm{m\,s^{-2}}$

Particle Q moves in a straight line, passing through the points with position vectors $(\mathbf{i} - \mathbf{j})$ metres and $(10\mathbf{i} + c\mathbf{j})$ metres.

P and Q are moving along parallel paths.

16 (a) Show that $c = -13$

[4 marks]

$P\ (-4i + 5j)$ $\quad$ $Q\ \ i - j$

$u = 0$ $\quad$ $10i + cj$

$a = 3i - 4j$

$v = 3ti - 4tj$ $\quad$ Q travelled $9i + (c+1)j$

if $i : j$ $\quad$ $9 : (c+1)j$ $\quad \div 3$

$3 : -4$ $\quad$ $3 : -4$

$\frac{c+1}{3} = -4$

$c + 1 = -12$

$c = -13$

16 (b) (i) Find an expression for the position vector of P at time t seconds.

[1 mark]

P started at $-4i + 5j$

$u = 0$

$v = 3ti - 4tj$ $\quad$ $s = \frac{1}{2}(u + v)t$

$a = 3i - 4j$ $\quad$ $= \frac{1}{2}(3ti - 4tj)t$

$= \frac{3t^2}{2}i - 2t^2 j$

Position at time t : $\left(\frac{3t^2}{2} - 4\right)i + (5 - 2t^2)j$

Do not write outside the box

16 (b) (ii) Hence, prove that the paths of P and Q are not collinear.

[3 marks]

Q goes from $(i - j)$ to $(9i - 13j)$

so moves $8i - 12j = 4(2i - 3j)$

P goes from $(-4i + 5j)$ to $(3t^2 - 4)i\ (5 - 2t^2 j)$

So moves $3t^2 i - 2t^2 j$

$= t^2(3i - 2j)$

$3i - 2j$ and $2i - 3j$ are not parallel and therefore not collinear

Turn over for the next question

Do not write outside the box

17 A particle is moving such that its position vector, **r** metres, at time t seconds, is given by

$$\mathbf{r} = \mathrm{e}^t \cos t\ \mathbf{i} + \mathrm{e}^t \sin t\ \mathbf{j}$$

Show that the **magnitude** of the acceleration of the particle, $a\,\mathrm{m\,s^{-2}}$, is given by

$r \to v \to a$

$$a = 2\mathrm{e}^t$$

Fully justify your answer.

[7 marks]

$i \quad e^t\cos t$

(1) $\frac{dr}{dt} = e^t\cos t - e^t(\sin t)\ i$ $\quad u = e^t \quad v = \cos t$

$\frac{du}{dt} = e^t \quad \frac{dv}{dt} = -\sin t$

$\frac{d^2r}{dt^2} = e^t\cos t - e^t\sin t$

$- \left[e^t\sin t + e^t\cos t \right]$ $\quad u_1 \quad v_1$

$e^t\sin t$

$\frac{du_1}{dt} = e^t \quad \frac{dv_1}{dt} \cos t$

$= -2e^t\sin t\ i$

(j) $r = e^t\sin t$

$\frac{dr}{dt} = e^t\sin t + e^t\cos t\ j$

$\frac{d^2r}{dt^2} = e^t\cancel{\sin t} + e^t\cos t + e^t\cos t - \cancel{e^t\sin t}$

$= 2e^t\cos t$

$|a| = \sqrt{(2e^t\sin t)^2 + (2e^t\cos t)^2}$

$= \sqrt{4e^{2t}\sin^2 t + 4e^{2t}\cos^2 t}$

$= \sqrt{4e^{2t}(\sin^2 t + \cos^2 t)}$

$= \sqrt{4e^{2t}}$

$= 2e^t$

Do not write outside the box

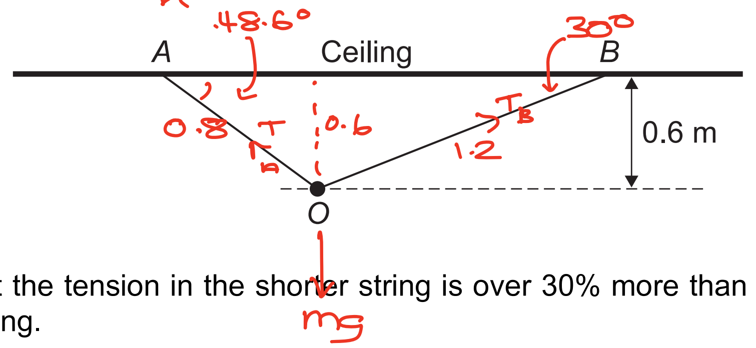
18 An object, O, of mass m kilograms is hanging from a ceiling by two light, inelastic strings of different lengths.

The shorter string, of length 0.8 metres, is fixed to the ceiling at A.

The longer string, of length 1.2 metres, is fixed to the ceiling at B.

This object hangs 0.6 metres directly below the ceiling as shown in the diagram.

$\sin^{-1}\left(\frac{3}{4}\right) = 48.6°$



$\sin^{-1}\left(\frac{1}{2}\right) = 30°$

18 (a) Show that the tension in the shorter string is over 30% more than the tension in the longer string.

[4 marks]

$\rightarrow$ $T_A \cos 48.6 = T_B \cos 30$

$T_A \times 0.6613 = T_B\ 0.866025$

$T_A = T_B \times \dfrac{0.866025}{0.6613}$

$T_A = T_B\ 1.3096$

which means T_A is over 30% more than T_B

Do not write outside the box

18 (b) The tension in the longer string is known to be $2g$ newtons.

Find the value of m.

[4 marks]

$mg = T_A \sin 48.6 + T_B \sin 30$

$T_B = 2g$

$mg = 2.62g \sin 48.6 + 2g \sin 30$ $T_A = 2.62g$

$mg = 29.05985 \ldots$

$m = 2.96529 \ldots g$

$m = 3.0\,kg$ (1dp)

Turn over for the next question

Do not write outside the box

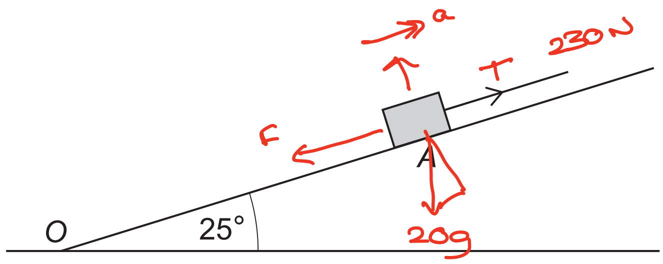
19 **In this question use $g = 9.8\,\text{m s}^{-2}$**

A rough wooden ramp is 10 metres long and is inclined at an angle of 25° above the horizontal. The bottom of the ramp is at the point O.

A crate of mass 20 kg is at rest at the point A on the ramp.

The crate is pulled up the ramp using a rope attached to the crate.

Once in motion, the rope remains taut and parallel to the line of greatest slope of the ramp.



19 (a) The tension in the rope is 230 N

The crate accelerates up the ramp at $1.2\,\text{m s}^{-2}$

Find the coefficient of friction between the crate and the ramp.

[7 marks]

$F_{net} = T - F - 20g \sin 25$

$R = 20g \cos 25$

$F_{net} = ma = 20 \times 1.2 = 24$

$24 = 230 - F - 82.833$

$F = 123.167$

$123.167 = \mu \times 20g \cos 25$

$\mu = 0.693$

Do not write outside the box

19 (b) (i) The crate takes 3.8 seconds to reach the top of the ramp.

Find the distance *OA*.

[3 marks]

$a = 1.2$

$t = 3.8$ $\quad s = ut + \frac{1}{2}at^2$

$u = 0$ $\quad = \frac{1}{2} \times 1.2 \times 3.8^2$

$s = ?$

$= 8.664$

ramp was 10m long

So $OA = 10 - 8.664 = 1.336\text{m}$

$= 1.34\text{ m}$ 3SF

19 (b) (ii) Other than air resistance, state **one** assumption you have made about the crate in answering part **(b)(i)**.

[1 mark]

The crate has been modelled as a particle

END OF QUESTIONS