Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Time 2 hours

Paper reference **8MA0/01**

Mathematics

Advanced Subsidiary

PAPER 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

1. Find

$$\int\left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right)\mathrm{d}x$$

giving your answer in simplest form.

(4)

$\int 8x^3 - \frac{3}{2}x^{-\frac{1}{2}} + 5$

$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$

$= 2x^4 - 3\sqrt{x} + 5x + c$

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2. $$f(x) = 2x^3 + 5x^2 + 2x + 15$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$. **(2)**

(b) Find the constants a, b and c such that

$$f(x) = (x + 3)(ax^2 + bx + c)$$

(2)

(c) Hence show that $f(x) = 0$ has only one real root. **(2)**

(d) Write down the real root of the equation $f(x - 5) = 0$ **(1)**

If $x+3$ is a factor $f(-3)=0$

$2(-3)^3 + 5(-3)^2 + 2(-3) + 15$

$= -54 + 45 - 6 + 15 = 0$

so $x+3$ is a factor

$$\begin{array}{r|l} & 2x^2 - x + 5 \\ x+3 & 2x^3 + 5x^2 + 2x + 15 \\ & 2x^3 + 6x^2 \\ & \quad -x^2 + 2x \\ & \quad -x^2 - 3x \\ & \qquad 5x + 15 \end{array}$$

$(x+3)(2x^2 - x + 5)$ $a=2$ $b=-1$ $c=5$

for $x-3$ to be the only root then the discriminant of $2x^2 - x + 5$ must be negative

$b^2 - 4ac = 1 - (4 \times 2 \times 5)$

$1 - 80$ is negative

If $x = -3$ is the root of $f(x)$

the root of $f(x-5)$ will be $-3+5 = 2$

3. The triangle PQR is such that $\overrightarrow{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 13\mathbf{i} - 15\mathbf{j}$

(a) Find $\overrightarrow{QR}$

(2)

(b) Hence find $|\overrightarrow{QR}|$ giving your answer as a simplified surd.

(2)

The point S lies on the line segment QR so that $QS : SR = 3 : 2$

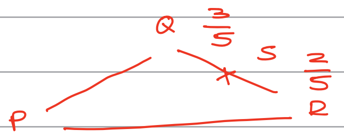
(c) Find $\overrightarrow{PS}$

(2)

$\overrightarrow{QR} = 10i - 20j$

$|\overrightarrow{QR}| = \sqrt{100 + 400}$

$= \sqrt{500} = 10\sqrt{5}$



$\overrightarrow{RS} = \frac{2}{5}\overrightarrow{RQ} = \frac{2}{5}(-10i + 20j)$

$= -4i + 8j$

$\overrightarrow{PS} = \overrightarrow{PR} + \overrightarrow{RS}$

$= 13i - 15j - 4i + 8j$

$= 9i - 7j$

4.

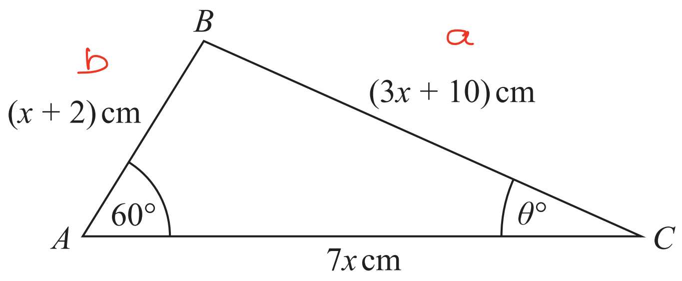


Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

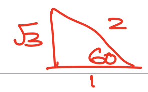
(3)

(ii) Hence find the value of x.

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)



Cosine rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

$(3x+10)^2 = (x+2)^2 + (7x)^2 - (2\times(x+2)\times(7x)\times\frac{1}{2})$

$9x^2+60x+100 = x^2+4x+4+49x^2 - [7x^2+14x]$

$9x^2+60x+100 = 43x^2-10x+4$

$0 = 34x^2-70x-96$

$0 = 17x^2-35x-48$

$0 = (17x+16)(x-3)$

$x = -\frac{16}{17}$ $\quad x = 3$

invalid as a length can't be negative

$\frac{5}{\sin\theta} = \frac{19}{\sin 60}$ $\qquad \sin\theta = \frac{5}{19}\times\sin 60$

$\theta = 13.2°$ (1dp)

(can't be 166.8 as too big for the triangle)

5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

(a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

(b) With reference to the model, interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find, according to the model,

(i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,

(ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

(d) State one reason why this may not be a realistic model in the long term.

(1)

$A = pq^t$

$\log_{10} A = 0.03t + 0.5$

$A = 10^{0.5} \times 10^{0.03t}$

$A = 3.162 \times 1.072^t$

$p = 3.162 \quad q = 1.072$

when $t = 0$ $A = p$ so p represents the initial mass of algae in the pond

q represents the rate that the algae increases each week

Question 5 continued

c) when $T=8$ $A = 3.162 \times 1.072^8$

$A = 5.522\ldots$ kg

$A = 5.5$kg (nearest 0.5kg)

c ii) $4 = 3.162 \times 1.072^t$

$\frac{4}{3.162} = 1.072^t$

$\log_{1.072}\left(\frac{4}{3.162}\right) = t$ $t = 3.381316\ldots$ weeks

$t = 3.38$ weeks (2dp)

d) This model is unrealistic as it predicts unlimited growth

6. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{2x}{9}\right)^8$$

giving each term in simplest form.

(4)

$$f(x) = \left(\frac{x-1}{2x}\right)\left(3 - \frac{2x}{9}\right)^8$$

(b) Find the coefficient of x^2 in the series expansion of f(x), giving your answer as a simplified fraction.

(2)

$\left(3 - \frac{2x}{9}\right)^8 = 3^8 + 3^7 \times 8 \times \left(\frac{-2}{9}\right)x + 3^6 \times \frac{8 \times 7}{2} \times \frac{(-2)^2}{9^2}x^2 + 3^5 \times \frac{8 \times 7 \times 6}{2 \times 3}\frac{(-2)^3}{9^3}x^3$

$= 6561 - 3888x + 1008x^2 - \frac{448}{3}x^3 + \ldots$

b) $6561 - 3888x + 1008x^2 - \frac{448x^3}{3}$ $\left(\frac{x-1}{2x}\right)$

coefficient x^2 will be found by multiplying ① $1008x^2 \times \frac{x-1}{2x}$ $1008x^2$

and ② $\frac{-448x^3}{3} \times \frac{(x-1)}{2x}$

① $\frac{1008x^3 - 1008x^2}{2x} = 504x^2 + 504x$

② $\frac{-\frac{448x^4}{3} + \frac{448x^3}{3}}{2x} = \frac{-224x^3}{3} + \frac{224x^2}{3}$

$504 + \frac{224}{3} = \frac{1736}{3}$

7. (a) Factorise completely $9x - x^3$

(2)

The curve C has equation

$$y = 9x - x^3$$

(b) Sketch C showing the coordinates of the points at which the curve cuts the x-axis.

(2)

The line l has equation $y = k$ where k is a constant.

Given that C and l intersect at 3 distinct points,

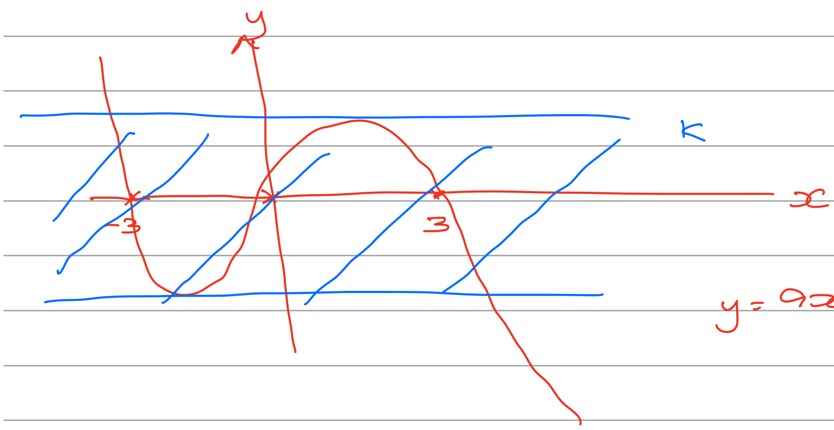
(c) find the range of values for k, writing your answer in set notation.

Solutions relying on calculator technology are not acceptable.

(3)

$x(9-x^2)$

$x(3-x)(3+x)$



$y = 9x - x^3$

Find Turning points

$\frac{dy}{dx} = 9 - 3x^2$ when $\frac{dy}{dx} = 0$ $3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}$

$x = \sqrt{3}$ $y = 9\sqrt{3} - 3\sqrt{3} = 6\sqrt{3}$

$x = -\sqrt{3}$ $y = -9\sqrt{3} - -3\sqrt{3} = -6\sqrt{3}$ $-\sqrt{3} \times -\sqrt{3} \times -\sqrt{3}$

$\{k \in \mathbb{R} : -6\sqrt{3} < k < 6\sqrt{3}\}$

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \qquad t \in \mathbb{R} \qquad t \geqslant 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm^2

(a) state the value of k.

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm^2
Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.
Give your answer in kg/cm^2 per minute to 3 significant figures.

(2)

when $t=0$ $\quad P = k + 1.4$

$2.2 = k + 1.4$

$0.8 = k$

b) $1 = 0.8 + 1.4e^{-0.5t}$

$\frac{0.2}{1.4} = e^{-0.5t}$

$\frac{\ln\left(\frac{1}{7}\right)}{-0.5} = t \qquad \frac{\ln(7)}{0.5} = t \qquad t = 2\ln(7) = 3.9 \text{ min}$ (1dp)

c) $\frac{dP}{dt} = -0.7e^{-0.05t}$ $\quad$ at $t = 2$

$\frac{dP}{dt} = -0.7e^{-0.1} = -0.258$ (3SF)

decreasing at a rate of 0.258 kg/cm^2 per min

9. (a) Given that $p = \log_3 x$, where $x > 0$, find in simplest form in terms of p,

(i) $\log_3\left(\frac{x}{9}\right)$

(ii) $\log_3\left(\sqrt{x}\right)$

(2)

(b) Hence, or otherwise, solve

$$2\log_3\left(\frac{x}{9}\right) + 3\log_3\left(\sqrt{x}\right) = -11$$

giving your answer as a simplified fraction.

Solutions relying on calculator technology are not acceptable.

(4)

i) $\log_3\left(\frac{x}{9}\right) = \log_3 x - \log_3 9$

$= \log_3 x - 2$

$= p - 2$

ii) $\log_3 \sqrt{x} = \log_3 x^{\frac{1}{2}}$

$= \frac{1}{2}\log_3 x \quad = \frac{p}{2}$

b) $2(p-2) + 3\left(\frac{p}{2}\right) = -11$

$2p - 4 + \frac{3p}{2} = -11$

$\frac{7p}{2} = -7 \qquad p = -2$

$\log_3 x = -2 \qquad 3^{-2} = x \qquad x = \frac{1}{9}$

10.

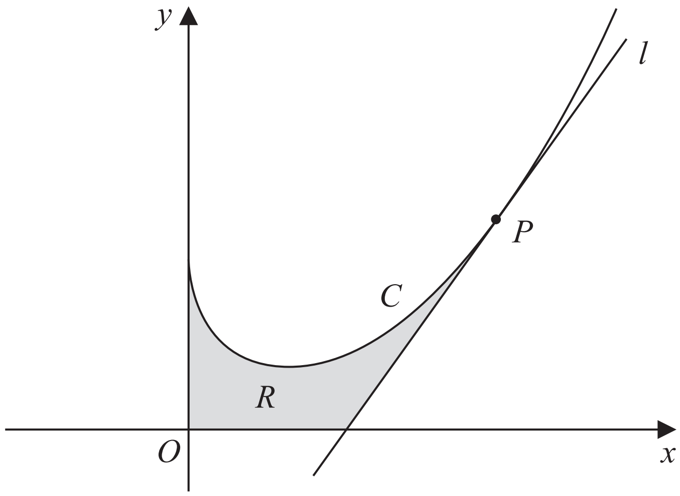


Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{1}{3}x^2 - 2\sqrt{x} + 3 \qquad x \geqslant 0$$

The point P lies on C and has x coordinate 4

The line l is the tangent to C at P.

(a) Show that l has equation

$$13x - 6y - 26 = 0$$

(5)

The region R, shown shaded in Figure 2, is bounded by the y-axis, the curve C, the line l and the x-axis.

(b) Find the exact area of R.

(5)

$x = 4 \quad y = \frac{16}{3} - 4 + 3 = \frac{13}{3} \qquad P = \left(4, \frac{13}{3}\right)$

Find gradient of curve

$\frac{dy}{dx} = \frac{2x}{3} - \frac{1}{\sqrt{x}} \qquad$ at $x = 4$ gradient $= \frac{8}{3} - \frac{1}{2} = \frac{13}{6}$

at $\left(4, \frac{13}{3}\right) \quad y = \frac{13}{6}x + c \qquad \frac{13}{3} = \frac{26}{3} + c$

$y = \frac{13x}{6} - \frac{13}{3} \qquad c = -\frac{13}{3}$

$13x - 6y - 26 = 0$

P 6 9 2 0 1 A 0 3 0 4 8

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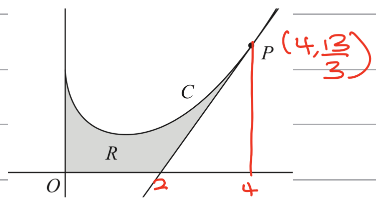
$2 \times \frac{2}{3}$ $\quad -2x^{\frac{1}{2}}$ $\quad \frac{4}{3}x^{\frac{3}{2}}$

Question 10 continued

b) $\displaystyle\int_0^4 \frac{1}{3}x^2 - 2\sqrt{x} + 3 = \left[\frac{x^3}{9} - \frac{4}{3}x^{\frac{3}{2}} + 3x\right]_0^4$

$= \left[\frac{64}{9} - \frac{32}{3} + 12\right] - \left[0\right]$

$= \frac{76}{9}$



$y = \frac{13}{6}x - \frac{13}{3}$ when $y = 0$

$\frac{13}{6}x = \frac{13}{3}$ $\quad x = 2$

area triangle $= \frac{1}{2} \times 2 \times \frac{13}{3} = \frac{13}{3}$

Area $R = \frac{76}{9} - \frac{13}{3} = \frac{37}{9}$

11.

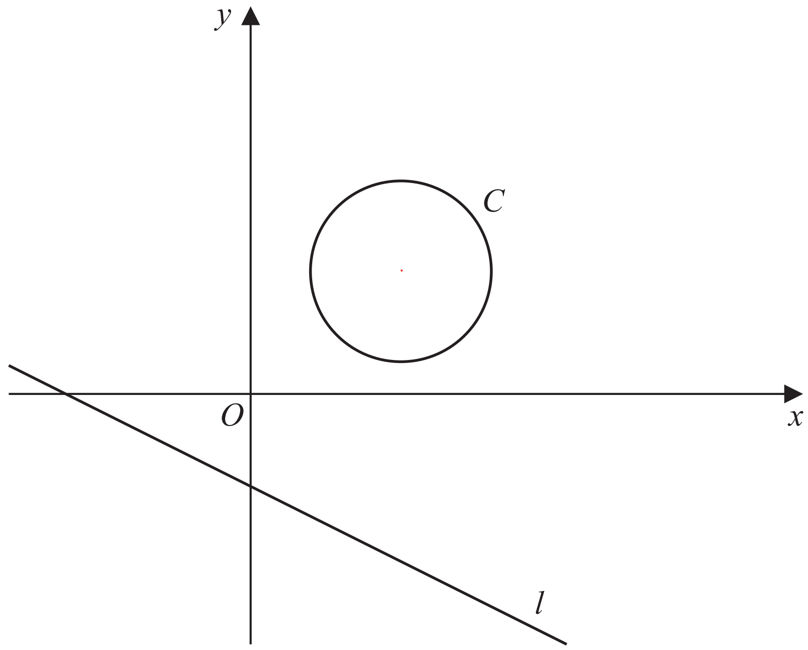


Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the radius of C.

(3)

(b) Find the shortest distance between C and l.

(5)

$x^2 - 10x + y^2 - 8y + 32 = 0$

$(x-5)^2 - 25 + (y-4)^2 - 16 + 32 = 0$

$(x-5)^2 + (y-4)^2 = 9$

Centre $(5, 4)$ radius 3

2) $2y + x + 6 = 0$

$2y = -x - 6$

$y = \frac{-x}{2} - 3$ gradient $= -\frac{1}{2}$

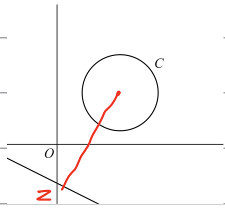
Question 11 continued

shortest distance where perpendicular line meets the centre

$y = 2x + c$ at $(5,4)$

$4 = 10 + c$

$-6 = c$ $\qquad \underline{y = 2x - 6}$



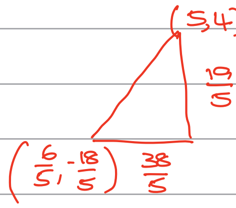
point on line where $y = 2x - 6$ meets $2y + x + 6 = 0$ (point z)

$2(2x - 6) + x + 6 = 0$

$5x - 6 = 0$

$x = \frac{6}{5}$ $\quad y = \frac{12}{5} - 6 = \frac{72 - 180}{30} = \frac{-108}{30} = \frac{-18}{5}$ $\qquad z = \left(\frac{6}{5}, \frac{-18}{5}\right)$

distance z to centre $= \sqrt{\left(\frac{19}{5}\right)^2 + \left(\frac{38}{5}\right)^2}$



$= \frac{19\sqrt{5}}{5}$

radius

$\frac{19\sqrt{5}}{5} - 3$ = shortest distance

12. A company makes drinks containers out of metal.

The containers are modelled as closed cylinders with base radius r cm and height h cm and the capacity of each container is $355\,\text{cm}^3$

The metal used

- for the circular base and the curved side costs 0.04 pence/cm^2
- for the circular top costs 0.09 pence/cm^2

Both metals used are of negligible thickness.

(a) Show that the total cost, C pence, of the metal for one container is given by

$$C = 0.13\pi r^2 + \frac{28.4}{r}$$

(4)

(b) Use calculus to find the value of r for which C is a minimum, giving your answer to 3 significant figures.

(4)

(c) Using $\frac{d^2C}{dr^2}$ prove that the cost is minimised for the value of r found in part (b).

(2)

(d) Hence find the minimum value of C, giving your answer to the nearest integer.

(2)

Cost of base $= \pi r^2 \times 0.04$

cost of top $= \pi r^2 \times 0.09$

$\pi r^2 h = 355$ cost of curved side $= 2\pi r \times \frac{355}{\pi r^2} \times 0.04$

$h = \frac{355}{\pi r^2}$

$= \frac{28.4}{r}$

r^{-1}

total cost $= 0.13\pi r^2 + \frac{28.4}{r}$

$\frac{dC}{dr} = 0.8168r - \frac{28.4}{r^2}$ at $\frac{dC}{dr} = 0$

$\frac{0.8168r^3 - 28.4}{r^2} = 0$

$0.8168r^3 = 28.4$ $\frac{28.4}{0.26\pi}$

$r = 3.26386\ldots$

$r = 3.26$ cm (3SF

$\frac{d^2C}{dr^2} = 0.8168 + \frac{56.8}{r^3}$ at $r = 3.26$ $\frac{d^2C}{dr^3} > 0$ $\therefore$ this is a minimum value

Question 12 continued

at $r = 3.26$ $C = 0.13\pi r^2 + \frac{28.4}{r}$

$C = 13.052$

cost = £13 nearest integer

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1-\sin\theta} \qquad \theta \neq (2n+1)90° \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90°$

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$$

giving your answers to one decimal place.

(5)

$\frac{1}{\cos\theta} + \frac{\sin}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta} \times \frac{\cos\theta}{\cos\theta}$

$= \frac{\cos\theta + \sin\theta\cos\theta}{\cos^2\theta}$ $\quad \sin^2 + \cos^2 = 1$

$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta} = \frac{\cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$= \frac{\cos\theta}{1-\sin\theta}$

$\frac{\cos 2x}{1-\sin 2x} = 3\cos 2x$ $\quad \cos 2x \neq 0$ so can $\div$ by $\cos 2x$

$1 = 3(1-\sin 2x)$

$1 = 3 - 3\sin 2x$

$3\sin 2x = 2$ $\quad \sin^{-1}\left(\frac{2}{3}\right) = 41.8$

$\sin 2x = \frac{2}{3}$

$2x = 41.81, 138.19, 401.81 \ldots$

$x = 20.9, 69.1,$

14. (i) A student states

"if x^2 is greater than 9 then x must be greater than 3"

Determine whether or not this statement is true, giving a reason for your answer.

(1)

(ii) Prove that for all positive integers n,

$$n^3 + 3n^2 + 2n$$

is divisible by 6

(3)

i) no as x could $= -4$ which is < 3 but $x^2 > 9$

$n^3 + 3n^2 + 2n = n(n^2 + 3n + 2)$
$= n(n+2)(n+1)$

this is 3 consecutive integers
any 3 consecutive integers must have at least one multiple of 2 and at least one multiple of 3 and therefore once 3 consecutive integers are multiplied it must produce a multiple of 6